Introduction to Nonlinear Dynamics
Indian Institute of Technology Madras
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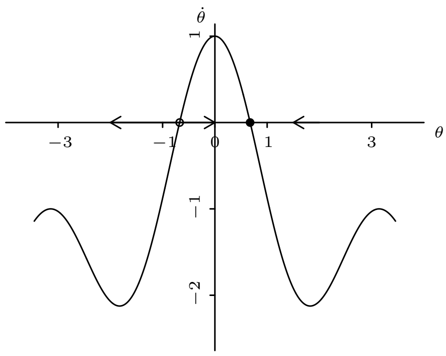
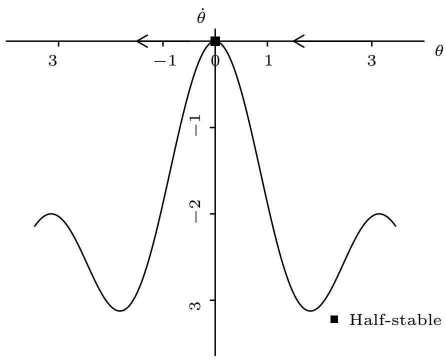
Solutions to Assignment 4 - Part 3

1 Nonuniform Oscillator

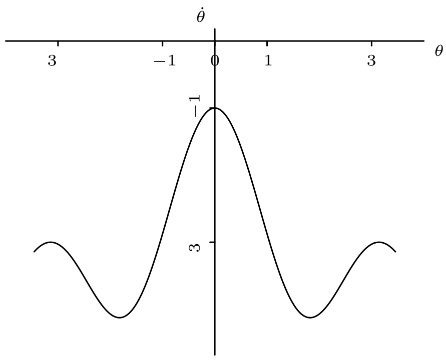
Next, we consider the case where $-2 < \mu < 0$. In this case, the negative root of the quadratic equation does not exist as it does not satisfy the condition $\cos\theta \geq -1$. Therefore, there is only one possible value of $\cos\theta$ corresponding to the positive root. This gives two fixed points, as shown in Figure 1.1a.
For $\mu = -2$, the negative root of the quadratic equation does not exist as it does not satisfy the condition $\cos\theta > -1$. There is only one possible value $\cos\theta = -1$. This leads to one fixed point at 0. This case is shown in Figure 1.1b.
The last case is $\mu < -2$. For this value of μ, the condition $-1 \leq \cos\theta \leq 1$ is violated. Hence, there are no fixed points. This can be seen in Figure 1.1c.
Therefore, the critical values are $\mu = -2, 0, 9/8$. As fixed points appear and disappear at each of these values via the existence of a half stable node, this is saddle-node bifurcation.



(a) $-2 < \mu < 0$, 1 stable and 1 unstable fixed point.



(b) $\mu = -2$, 1 half stable fixed point.



(c) $\mu < -2$, no fixed points.

Figure 1.1: Phase portraits for $\dot{\theta} = \mu + \cos\theta + \cos 2\theta$, for various values of parameter μ.