1 Saddle-Node bifurcation

1) Consider the system $\dot{x} = 1 + rx + x^2$.
   a) Sketch the qualitatively different vector fields that occur as $r$ is varied.
   b) Determine the critical value of $r$ at which a saddle-node bifurcation occurs.

2 Transcritical bifurcation

1) Show that the system $\dot{x} = x(r-e^x)$ undergoes a transcritical bifurcation. Determine
   the value of $r$ at which the bifurcation occurs and sketch the bifurcation diagram
   of fixed points $x^*$ vs $r$.

3 Pitchfork bifurcation

1) Consider the following system $\dot{x} = rx - 4x^3$.
   a) Sketch the qualitatively different vector fields that occur as $r$ is varied.
   b) Determine the critical value of $r$ at which a pitchfork bifurcation occurs.
   c) Classify the bifurcation as supercritical or subcritical.

4 Identify the bifurcation

1) In each of the following systems, determine the value of $r$ at which a bifurcation
   occurs, and classify the bifurcation as saddle-node, transcritical, supercritical
   pitchfork or subcritical pitchfork.
   a) $\dot{x} = 5 - re^{-x^2}$
   b) $\dot{x} = r - 3x^2$