Assignment 2

Due: October 8, 2015, 23:30 (IST)

1 Existence and uniqueness

1) A particle travels on the half-line \( x \geq 0 \) with a velocity given by \( \dot{x} = -x^c \), where \( c \) is real and constant.
   a) Find all values of \( c \) such that the origin \( x = 0 \) is a stable fixed point.
   b) Now assume that \( c \) is chosen such that \( x = 0 \) is stable. Can the particle ever reach the origin in a finite time? Specifically, how long does it take for the particle to travel from \( x = 1 \) to \( x = 0 \), as a function of \( c \)?

2 Impossibility of oscillations

1) [No periodic solutions to \( \dot{x} = f(x) \)] Here's an analytical proof that periodic solutions are impossible for a vector field on a line. Suppose on the contrary that \( x(t) \) is a nontrivial periodic solution, i.e., \( x(t) = x(t+T) \) for some \( T > 0 \), and \( x(t) \neq x(t+s) \) for all \( 0 < s < T \). Derive a contradiction by considering \( \int_{t}^{t+T} f(x) \frac{dx}{dt} dt \).

3 Potentials

1) For each of the following vector fields, plot the potential function \( V(x) \) and identify all the equilibrium points and their stability.
   a) \( \dot{x} = x(1-x) \)
   b) \( \dot{x} = 2 + \sin x \)
   c) \( \dot{x} = r + x - x^3 \), for various values of \( r \).