Exercises

1. Complete the derivation of equation for constant x circles of a Smith chart.

2. Complete the derivation of equation for constant r circles of a Smith chart.

3. Locate (i) short circuit impedance, (ii) open circuit impedance, (iii) matched impedance points on Smith chart.

Assignment #3

1. It is desired to obtain a normalized reactance of +j0.5 using an open circuited lossless transmission line of length $l$. If the frequency of operation is 2 GHz, the minimum length $\ell$ in terms of wavelength is

$$\frac{Z_{oc}}{Z_0} = -j \cot \beta \ell = j0.5$$

$$\beta \ell = \cot^{-1} -0.5 = -1.1071$$

$$\frac{l}{\lambda} = -0.1762 \text{ or } 0.0796$$

[For problems 2-7 use Smith Chart]

2. A transmission line having characteristic impedance of 50 ohms is terminated in a complex load impedance of 50+j100 ohms. The operating frequency on the line is 100 MHz. The line is 0.125\(\lambda\) long. The real part of the input impedance seen looking into the transmission line is (in ohms)

$$Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0}{Z_0 + jZ_L} \right] = 50(1 - 2j)$$

3. For problem 2, the imaginary part of the input impedance seen looking into the transmission line is (in ohms)

$$Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0}{Z_0 + jZ_L} \right] = 50(1 - 2j)$$

4. If the load in problem 2 is replaced by a short circuit, the new value of real part of the complex input impedance is (in ohms)

$$Z_{in} = jZ_0 \tan \beta \ell = j50$$

5. If the load in problem 2 is replaced by a short circuit, the new value of imaginary part of the complex input impedance is (in ohms)

$$Z_{in} = jZ_0 \tan \beta \ell = j50$$

6. If the load in problem 2 is replaced by an open circuit, the new value of real part of the complex input admittance is (mhos)

$$Z_{in} = -jZ_0 \cot \beta \ell = -j50$$
7. If the load in problem 2 is replaced by an open circuit, the new value of imaginary part of the complex input admittance is (in mhos)

\[ Y_{in} = \frac{1}{Z_{in}} = j0.02 \]

\[ Z_{in} = -jZ_0 \cot \beta l = -j50 \]

\[ Y_{in} = \frac{1}{Z_{in}} = j0.02 \]

8. Consider a transmission line of length \( \ell = 0.1\lambda \). The transmission line has a characteristic impedance of 50 ohms. The magnitude of \( Z_{12} \) parameter of the T-equivalent circuit of the transmission line is (in ohms)

\[ Z_{12} = -jZ_0 \csc \beta l = -j85.065 \]

[For problems 9-17 use Smith chart]

9. If the measured reflection coefficient on a line is 0.4+j0.2, the corresponding normalized load impedance (real part only) is

\[ Z_L' = \frac{1 + \Gamma_L}{1 - \Gamma_L} = \frac{1.4 + j0.2}{0.6 + j0.2} = 2 + j1 \]

10. For problem 10, the normalized load impedance (imaginary part only) is

\[ Z_L' = \frac{1 + \Gamma_L}{1 - \Gamma_L} = \frac{1.4 + j0.2}{0.6 + j0.2} = 2 + j1 \]

11. Given load impedance of 100+j200 ohms connected to a transmission line of 50 ohms characteristic impedance, the magnitude of the reflection coefficient is

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.76 + j0.32 \]

\[ |\Gamma_L| = 0.824 \]

12. Given load impedance of 30+j25 ohms connected to a transmission line of 50 ohms characteristic impedance, the nearest location from the load at which the impedance is real is (in terms of wavelength \( \lambda \))

\[ \frac{l}{\lambda} = 0.154 \]

13. For problem 12, the nearest location from the load at which the impedance is real and maximum is (in terms of wavelength \( \lambda \))

\[ \frac{l}{\lambda} = 0.154 \]
14. For problem 12, the nearest location from the load at which the impedance is real and minimum is (in terms of wavelength $\lambda$)

$$\frac{l}{\lambda} = 0.4046$$

15. For problem 12, the location of first voltage maxima from the load is ((in terms of wavelength $\lambda$)

$$\frac{l}{\lambda} = \frac{-2\pi - \phi_L}{4\pi} = -0.4046$$

16. The VSWR patterns measured along a transmission line terminated in unknown load is shown in figure here. From the data given in figure, the real part of the unknown complex load is (in ohms) [Take characteristic impedance of 50 ohms]

\[ V_{\text{max}} = 4, V_{\text{min}} = 2.5 \]

\[ S = \frac{V_{\text{max}}}{V_{\text{min}}} = 1.6 \]

\[ |\Gamma_L| = \frac{S - 1}{S + 1} = 0.2308 \]

\[ \Gamma_L = 0 + j0.2308 \]

\[ Z_L = Z_0(0.8989 + j0.4383) \]

\[ Z_L = 44.9425 + j21.9127 \]

17. The VSWR patterns measured along a transmission line terminated in unknown load is shown in figure here. From the data given in figure, the imaginary part of the unknown complex load is (in ohms) [Take characteristic impedance of 50 ohms]
\[ S = \frac{V_{\text{max}}}{V_{\text{min}}} = 1.6 \]
\[ |\Gamma_L| = \frac{S - 1}{S + 1} = 0.2308 \]
\[ \Gamma_L = 0 + j0.2308 \]
\[ Z_L = Z_0 (0.8989 + j0.4383) \]
\[ Z_L = 44.9425 + j21.9127 \]

18. A quarter wave transformer (QWT) is used to match a section of the transmission line of characteristic impedance 75 ohms with load of 100 ohms. The characteristic impedance of the QWT is (in ohms)

\[ R_0 = \sqrt{R_L R_G} = 86.6025 \]

19. A certain generator of 100 volts, 25 ohm internal impedance is to be matched to a load such that maximum power is delivered to the load. The required load resistance to achieve this is (in ohms)

\[ P_A = \frac{V_G^2}{R_s} = 400W \]
\[ P_L^{\text{max}} = \frac{1}{2} P_A = 200W \]
\[ R_L = \frac{V_G^2}{4 P_L^{\text{max}}} = 12.5 \text{ ohms} \]

20. In problem 19, the amount of power delivered to the load is (in Watts)

\[ P_A = \frac{V_G^2}{R_s} = 400W \]
\[ P_L^{\text{max}} = \frac{1}{2} P_A = 200W \]
\[ R_L = \frac{V_G^2}{4 P_L^{\text{max}}} = 12.5 \text{ ohms} \]