Week-2 Exercises

1. Derive the differential equation showing the variation of voltage phasor along the transmission line. Module #6.
2. Conversion of attenuation constant units from Neper/meter to dB/m. Module #6
3. Verify the expression of \( V_0^+ \) discussed in Module #7.
4. Verify the expression of the voltage on transmission line after substituting the reflection coefficient in polar form in Module #8.
5. Verify the expressions of \( V_{\text{max}} \) and \( V_{\text{min}} \) discussed in Module #8
6. Find the location of the first maxima when the transmission line is terminated with pure capacitance. Module #8
7. By applying KVL, verify if the T-Network discussed in Module #9 is exact equivalent of the matrix relationship shown in the video.
8. For the problem of interfacing two IC’s with T-lines discussed in Module #9 calculate the value of \( Z_{12} \)
9. The characteristic impedance of a line is purely real. Does this mean that the line is lossless? (Yes/No).
10. The SWR on a transmission line terminated in its own impedance is unity (Yes/No).
11. The characteristic impedance of a transmission line is complex at the operating frequency. The line is terminated in its own characteristic impedance. The SWR on the line is (unity/infinity/zero).
12. One can obtain any value of reactance by changing the length of a lossless transmission line terminated in short circuit. Suppose the length of the line is to remain fixed. How can different reactances be generated?
13. Sketch the standing wave pattern on a lossless transmission line when terminated in (a) open circuit, (b) short circuit, (c) pure inductor, (d) its own characteristic impedance.
14. On a lossless transmission line terminated in an unknown load, the SWR depends on frequency of operation (Yes/No).
15. What length of transmission line should be used to convert a pure inductance load to a pure capacitive reactance?
16. Repeat exercise 7 if the load is pure inductance and we desire to convert to pure capacitance.
17. [Difficult] Determine the line voltage on a lossy transmission line at a distance of \( \lambda/8 \) from the load if attenuation coefficient = 0.01/m and propagation coefficient = 31.415 rad/m. Assume that the line is terminated in open circuit load. The incident voltage amplitude is 100 V.
18. [Difficult] What is the incident voltage amplitude on a lossless transmission line of length terminated in a short circuit?
Assignment #2

1. Given the primary constants $L$ and $C$ of a transmission line to be 2 nH/m and 13 pF/m, the phase velocity on the line is (in G m/sec) where $G=10^9$

Ans. $U_p = 1/\sqrt{LC} = 6.2$

2. For problem 1, the characteristic impedance $Z_0$ of the line is (in ohms)

Ans:
\[ Z_0 = \sqrt{\frac{L}{C}} = 12.4 \text{ ohms} \]

3. The characteristic impedance of a transmission line is 50 ohms and the phase velocity is 0.67$c$, where $c$ is the speed of light in vacuum. The primary constant $C$ is (Ans. 100 pF/m)

4. For problem 3, the primary constant $L$ is (Ans. 250 nH/m)

5. A transmission line ($Z_0=100$ ohms and $u_p=2\times10^8$ m/s) is terminated in pure capacitance of $C = 20$ pF. The operating frequency of the line is 2 GHz. The real part of load reflection coefficient is

\[ Z_L = \frac{1}{j\omega C} = -3.97j \]
\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.9968 - 0.0792j \]
\[ |\Gamma_L| \sim 1 = 0.999 \]

6. For problem 5, the position of first voltage minima from the load (z=0) is at (in meters)

Ans:
\[ \phi_L = \tan^{-1} \frac{0.0792}{0.9968} = 0.07928 \]
\[ Z_{min} = \frac{-\pi - \phi_L}{2\beta} = -0.038 \]

7. For problem 5, the first voltage maxima from the load occurs at (in meters)

\[ \phi_L = \tan^{-1} \frac{0.0792}{0.9968} = 0.07928 \]
\[ Z_{min} = \frac{2\pi - \phi_L}{2\beta} = -0.076 \]
8. A transmission line having characteristic impedance of 50 ohms is terminated in a complex load impedance of 50+j100 ohms. The operating frequency on the line is 100 MHz. The line is 0.125λ long. The input impedance seen looking into the transmission line is (ohms)

And: \( \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \frac{\pi}{4} \)

\[ Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0}{Z_0 + jZ_L} \right] = 50(1 - 2j) \]

9. If the load in problem 8 is replaced by a short circuit, the new value of input impedance is (in ohms)

Ans: \( Z_{in} = j50 \)

10. A certain uniform lossless transmission line is terminated in complex load \((1+j0.25)Z_0\). The voltage maxima, normalized to incident voltage amplitude, on the line is

Ans:

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.01538 + 0.12307j \]

\[ \frac{V_{max}}{V_0^+} = 1 + |\Gamma_L| = 1.12 \]

11. The current minima, normalized to incident current amplitude, for the transmission line problem in 10 is

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.01538 + 0.12307j \]

\[ \frac{V_{min}}{V_0^+} = 1 - |\Gamma_L| = 0.88 \]

12. Two ICs are interconnected by a 1 cm long microstrip line. The driver IC sends a pulse of 10 V amplitude and 2 ns risetime, 0.8 ns fall time, and 1 ns width down the line to the load IC. Using the thumb rule discussed in lectures, should the microstrip line be treated as a transmission line?

Yes

No

Information about phase velocity is required to answer this question

13. For the problem above, highest frequency of interest (according to thumb rule of 0.25/tr) is

Ans:

\( f_{max} = 0.3125 \text{ GHz} \)
14. The standing wave ratio on a lossless line is 2. The magnitude of the reflection coefficient on the line is

\[ \Gamma_L = \frac{S - 1}{S + 1} = \frac{1}{3} \]

15. For the transmission line problem of 10, SWR on the line is

\[ S = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = 1.28 \]

16. The voltage on a transmission line is given by \( V(t) = 150\cos(7.5 \times 10^{10}t - 500z) \). The frequency, phase velocity, and wavelength are

Ans:

\[ \omega = 7.5 \times 10^{10}; \quad f = \frac{\omega}{2\pi} = 1.19366 \times 10^{10} \text{ Hz} = 11.94 \text{ GHz} \]

\[ \beta = 500; \quad U_p = \frac{\omega}{\beta} = 0.375 \times 10^9 \text{ m/sec} \]

\[ \lambda = \frac{c}{f} = 0.025 \text{ m} \]

17. An infinite length transmission line is lossless and has a characteristic impedance of 50 ohms. A signal generator with internal impedance of 100+j50 ohms and amplitude of 100 volts rms is connected to the input of the transmission line at \( z = 0 \). The sending end voltage, sending end current, and input impedance are

Ans:

\[ V = 10(1-3j) \]

\[ I = 0.2(1-3j) \]

\[ Z_{in} = 50 \]

18. In the problem 18, if the transmission line is terminated at \( z = \lambda/10 \) with a load 100 ohms, the sending end voltage is

\[ Z_{in} = 49.104-j35.025 \]

\[ V = 30.268-26.53j \]

19. A transmission line has \( R = 0.053 \text{ ohms/m}, \quad L = 0.62 \text{ \mu H/m}, \quad C = 40 \text{ pF/m} \). The characteristic impedance of the line at 900 MHz is

Ans:

\[ Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = 124.5 - j9.4e - 4 \]
20. For the transmission line in problem 20, the attenuation coefficient is
\[ \alpha = 0.0002 + 28.1610i \]

21. Two ICs are interconnected by a 1 cm long microstrip line. The driver IC sends a pulse of 10 V amplitude and 0.1 ns risetime, 0.1 ns fall time, and 1 ns width down the line to the load IC. Using the thumb rule discussed in lectures, should the microstrip line be treated as a transmission line?

- Yes
- No
- Information about phase velocity is required to answer this question