Assignment 11
The due date for submitting this assignment has passed.

Due on 2020-06-15, 23:59 EST.

1. A Kalman filter is
   a) a FIR filter of fixed length implemented recursively
   b) a BPF
   c) an order recursive filter
   d) a signal model based linear filter

   Answer: c) signal model based linear filter

2. Consider a signal $x(t)$, signal $y(t) = x(t) - 1 + W(t)$ and the noisy observation $y'(t)$ where $W(t)$ and $F(t)$ are white noises and $y'(t)$ is independent of $y(t)$ and $F(t)$. Then the Kalman innovation signal $F(t)$ is given by

$$ F(t) = y(t) - E[y'(t) | y(t-1)] $$

Answer: $F(t) = y(t) - y(t-1)$

3. Consider the signal $x(t)$ and the observation model in Q2.1 and the scalar Kalman filter to estimate $x(t)$. If the variance of the observation noise $W(t)$ is 2.5, then the Kalman gain $K(t)$ and the minimum mean-square estimation error $P(t)$ are given by

$$ K(t) = P(t-1)(x(t) - E[x(t)]) $$
$$ P(t) = E[(x(t) - E[x(t)])] $$

Answer: $K(t) = P(t-1)x(t)$, $P(t) = E[(x(t) - E[x(t)])]$

4. Consider the problem of estimating an unknown constant $x$ in the presence of white noise of variance $W(t)$ by a scalar Kalman filter. Over a signal the signal $F(t)$ is given by

$$ E[(x(t) - E[x(t)])] = W(t) $$

Answer: $E[(x(t) - E[x(t)])] = W(t)$

5. The number of states variables to represent an ARM962C signal is

Answer: 5

6. Consider the state space model:

$$ [x(t)] = A[t] x(t-1) + B[t] $$

Observation equation:

$$ y(t) = C[t] x(t-1) + E[t] $$

Both $W(t)$ and $F(t)$ are zero-mean white noise. Assuming $A(t)$ is given by

$$ \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} $$

The Kalman state estimate $\hat{x}(t)$ is equal to

$$ \begin{bmatrix} \hat{x}(t) \\ \hat{y}(t) \\ \vdots \\ \hat{y}(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \\ \vdots \\ y(t) \end{bmatrix} $$

Answer: $\begin{bmatrix} \hat{x}(t) \\ \hat{y}(t) \\ \vdots \\ \hat{y}(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \\ \vdots \\ y(t) \end{bmatrix}$