Unit 5 - Week 3
Assignment 3

1. What is the value of the following integral?

\[ \int_0^1 x^2 \, dx \]

2. Evaluate the following definite integral:

\[ \int_{-1}^{2} \frac{1}{x^2} \, dx \]

3. Consider the function defined by:

\[ f(x) = \begin{cases} 
2x^2 + 1 & \text{if } x < 0 \\
3x - 1 & \text{if } x \geq 0 
\end{cases} 
\]

Find the value of \( f(2) \) and \( \int_{-1}^{2} f(x) \, dx \).

4. Sketch the graph of the following function:

\[ f(x) = \sin(x) + \cos(x) \]

5. A system of linear equations has the following matrix representation:

\[ \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix} \]

Find the values of \( x \) and \( y \).

6. Let \( f(x) = x^2 - 4x + 3 \). Find the critical points of \( f(x) \) and determine whether each is a local maximum, local minimum, or neither.

7. Consider the following differential equation:

\[ \frac{dy}{dx} = 2y \]

Solve the differential equation and find the general solution.

8. Suppose \( f(x) \) is a pdf of a continuous random variable. If \( f(x) \) is symmetric about the origin, then its average value is zero. If \( f(x) \) is not symmetric about the origin, then its average value is not zero.

9. Find the expected value of a random variable with the following probability distribution:

\[ P(X = x) = \begin{cases} 
\frac{1}{3} & \text{if } x = 1 \\
\frac{1}{3} & \text{if } x = 2 \\
\frac{1}{3} & \text{if } x = 3 
\end{cases} \]

10. A random variable \( X \) has the following probability distribution:

\[ P(X = x) = \begin{cases} 
\frac{1}{4} & \text{if } x = 1 \\
\frac{1}{4} & \text{if } x = 2 \\
\frac{1}{4} & \text{if } x = 3 \\
\frac{1}{4} & \text{if } x = 4 
\end{cases} \]

Find the mean and variance of \( X \).