

# Unit 12 - Week 10

## Course outline

How does an NPTEL online course work?

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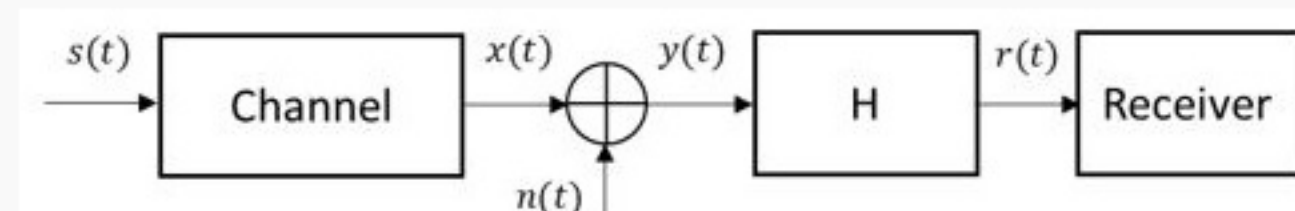
Assignment Solution

## Assignment 10

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2020-04-08, 23:59 IST.

1) Consider a communication system as shown below, where the transmitted signal  $s(t)$  propagates through a channel and is corrupted by noise  $n(t)$ . It is then acted upon by an unknown deterministic function  $H$ . 1 point



If the probabilities of error obtained by analyzing  $y(t)$  and  $r(t)$  are  $P_{e1}$  and  $P_{e2}$ , respectively, then the relation between  $P_{e1}$  and  $P_{e2}$  is

- $P_{e1} > P_{e2}$
- $P_{e1} < P_{e2}$
- $P_{e1} = P_{e2}$
- dependent on  $H$

No, the answer is incorrect. Score: 0

Accepted Answers:  $P_{e1} = P_{e2}$

2) The transmitter in a communication system sends antipodal vectors  $\vec{b}$  and  $-\vec{b}$  to transmit binary symbols '0' and '1', respectively, through a channel with additive white Gaussian noise having zero mean and variance  $\mu^2$ . It is given that  $\vec{b} = \{\mu, -2\mu, -3\mu, 3\mu, -4\mu, 5\mu\}$  and the symbol '0' is thrice as likely as the symbol '1'. The system was operating satisfactorily with an acceptable value of bit error rate. Now, due to some attenuation in the system, the signal to noise ratio reduces to half of its previous value. Determine the total error probability in the new case. Here  $Q(\cdot)$  denotes the  $Q$ -function. 1 point

- $\frac{1}{4} [3Q(5.75) + Q(5.56)]$
- $\frac{1}{4} [Q(5.75) + 3Q(5.56)]$
- $\frac{1}{4} [3Q(4.75) + Q(4.56)]$
- $\frac{1}{4} [Q(4.75) + 3Q(4.56)]$

No, the answer is incorrect. Score: 0

Accepted Answers:  $\frac{1}{4} [3Q(5.75) + Q(5.56)]$

3) The probability of error for various constellation schemes can be given as  $Q\left(\sqrt{\frac{\eta_p E_b}{2N_0}}\right)$  where  $\eta_p$  for OOK, FSK, BPSK, and binary simplex schemes, respectively, are 1 point

- 4, 2, 2, 4
- 4, 4, 2, 2
- 2, 2, 4, 4
- 2, 4, 4, 2

No, the answer is incorrect. Score: 0

Accepted Answers: 2, 2, 4, 4

4) If we achieve probability of error of  $10^{-8}$  at  $E_b/N_0 = 15$  dB for OOK scheme, then at what  $E_b/N_0$  can we achieve the same probability of error for BFSK1 scheme? 1 point

- 18 dB
- 15 dB
- 12 dB
- 9 dB

No, the answer is incorrect. Score: 0

Accepted Answers: 15 dB

5) Consider an OOK system, where the received signals for bit '1' and bit '0' are as follows 2 points

$$y(t) = \begin{cases} s(t) + n(t), & \text{bit 1 sent} \\ n(t), & \text{bit 0 sent} \end{cases}$$

where  $s(t) = I_{[0,1]}(t) + (2 - t)I_{[1,2]}(t)$  and  $n(t)$  is an additive white Gaussian noise with one-sided power spectral density of  $N_0$ . The received signal is passed through a filter with an impulse response  $h_1(t) = s(-t)$  and  $h_2(t) = I_{[-2,0]}(t)$  followed by a sampler at  $t_0$ . Here  $I_{[a,b]}(t)$  denotes a rectangular pulse of unit amplitude spanning from  $t = A$  to  $t = B$ . If a sample at  $t_0 = 0$  is used, then the BER at the output of the sampler for  $h_1(t)$  and  $h_2(t)$  is

- $Q\left(\sqrt{\frac{E_b}{2N_0}}\right), Q\left(\sqrt{\frac{3E_b}{2N_0}}\right)$
- $Q\left(\sqrt{\frac{E_b}{N_0}}\right), Q\left(\sqrt{\frac{3E_b}{2N_0}}\right)$
- $Q\left(\sqrt{\frac{E_b}{2N_0}}\right), Q\left(\sqrt{\frac{3E_b}{4N_0}}\right)$
- $Q\left(\sqrt{\frac{E_b}{N_0}}\right), Q\left(\sqrt{\frac{3E_b}{4N_0}}\right)$

No, the answer is incorrect. Score: 0

Accepted Answers:  $Q\left(\sqrt{\frac{E_b}{N_0}}\right), Q\left(\sqrt{\frac{3E_b}{4N_0}}\right)$

### GROUP-A

A PAM signaling scheme transmits values ' $a_1$ ' and ' $a_0$ ' for symbols '1' and '0', respectively, through a channel with additive white Gaussian noise having zero mean and variance  $2\alpha$ , such that  $a_0 + a_1 = -2$  and  $a = a_0 - m$  where  $m$  is the mean of  $a_1$  and  $a_0$ . The prior of symbol '1' is  $P_H(1)$  and that of symbol '0' is  $P_H(0)$ . These are related as  $P_H(1) = \eta P_H(0)$  where  $H$  denotes the hypothesis and  $\eta$  is a constant. The following two transmitters are available:  $T_1$  which provides  $\eta = 1$  and  $T_2$  which provides  $\eta = \sum_{n=0}^{\infty} (1/n!)$ . If the received random variable is  $X$  (having numerical value  $x$ ), then answer the following questions.

6) What is the decision rule for detection if  $T_1$  is used? 2 points

- $\begin{matrix} H = 0 \\ x \lesseqgtr 0 \\ H = 1 \end{matrix}$
- $\begin{matrix} H = 0 \\ x \lesseqgtr -1 \\ H = 1 \end{matrix}$
- $\begin{matrix} H = 0 \\ x \lesseqgtr 1 \\ H = 1 \end{matrix}$
- $\begin{matrix} H = 0 \\ x \lesseqgtr 0 \\ H = 1 \end{matrix}$

No, the answer is incorrect. Score: 0

Accepted Answers:  $\begin{matrix} H = 0 \\ x \lesseqgtr -1 \\ H = 1 \end{matrix}$

7) What is the decision rule for detection if  $T_2$  is used? 1 point

- $\begin{matrix} H = 0 \\ x \lesseqgtr 0 \\ H = 1 \end{matrix}$
- $\begin{matrix} H = 0 \\ x \lesseqgtr -1 \\ H = 1 \end{matrix}$
- $\begin{matrix} H = 0 \\ x \lesseqgtr 1 \\ H = 1 \end{matrix}$
- $\begin{matrix} H = 0 \\ x \lesseqgtr 0 \\ H = 1 \end{matrix}$

No, the answer is incorrect. Score: 0

Accepted Answers:  $\begin{matrix} H = 0 \\ x \lesseqgtr 0 \\ H = 1 \end{matrix}$

8) Given  $\alpha = 2$ , what is the total probability of error if  $T_1$  is used? 1 point

- $Q(0.25)$
- $Q(0.5)$
- $Q(1)$
- $Q(1.5)$

No, the answer is incorrect. Score: 0

Accepted Answers:  $Q(1)$

9) Given  $\alpha = 2$ , what is the total probability of error if  $T_2$  is used? 1 point

- $0.5[Q(1) + Q(0.5)]$
- $0.5[Q(0.5) + Q(1.5)]$
- $0.27 \times Q(1) + 0.73 \times Q(0.5)$
- $0.27 \times Q(0.5) + 0.73 \times Q(1.5)$

No, the answer is incorrect. Score: 0

Accepted Answers:  $0.27 \times Q(0.5) + 0.73 \times Q(1.5)$

### GROUP-B

A repetition coder which repeats 3 bits is used with binary on-off keying scheme. We assume  $a^2/2N_0$  is 15 in dB scale, where  $a$  is the amplitude of each bit. Answer the following questions

10) The symbol error rate if hard decision is used per bit with majority decoding is approximately 2 points

- $3 \times 10^{-12}$
- $3 \times 10^{-16}$
- $3 \times 10^{-8}$
- $3 \times 10^{-11}$

No, the answer is incorrect. Score: 0

Accepted Answers:  $3 \times 10^{-16}$

11) The symbol error rate if ML detection is used for 3 bits together is approximately 1 point

- $1 \times 10^{-22}$
- $9.2 \times 10^{-85}$
- $1 \times 10^{-85}$
- $9.2 \times 10^{-22}$

No, the answer is incorrect. Score: 0

Accepted Answers:  $1 \times 10^{-22}$

12) The symbol error rate if ML detection is used for  $n$  bits together is 1 point

- $Q\left(\sqrt{\frac{nE_b}{2N_0}}\right)$
- $Q\left(\sqrt{\frac{2nE_b}{N_0}}\right)$
- $Q\left(\sqrt{\frac{nE_b}{4N_0}}\right)$
- $Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

No, the answer is incorrect. Score: 0

Accepted Answers:  $Q\left(\sqrt{\frac{E_b}{N_0}}\right)$