Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis

Week 5 Assignment

April 3, 2017

1. $G_0(z)$ and $G_1(z)$ are low-pass and high pass synthesis side filters respectively and $H_0(z)$ and $H_1(z)$ are low-pass and high-pass analysis side filters respectively. Identify the correct relations between them along with the correct property.

   (a) $G_0(z) = H_1(-z)$, $G_1(z) = -H_0(-z)$, Alias Cancellation
   (b) $G_0(z) = H_1(-z)$, $G_1(z) = -H_0(-z)$, Power Complementarity
   (c) $G_0(z) = H_1(z)$, $G_1(z) = -H_0(-z)$, Alias Cancellation
   (d) $G_0(z) = H_1(z)$, $G_1(z) = -H_0(-z)$, Power Complementarity

   Solution: (a). Due to alias cancellation we require the following relation to hold true $T_1(z) = G_0(z)H_0(-z) + G_1(z)H_1(-z) = 0$. Hence if $G_0(z) = H_1(-z)$, then we get the relation $G_1(z) = -H_0(-z)$

2. Which of the following Z-Transforms satisfy the property: $H_0(e^{-j\omega}) = \overline{H_0(e^{j\omega})}$

   (a) $i + z^{-1}$
   (b) $1 + z^{-1}$
   (c) $\frac{1}{\sqrt{2}}(i + z^{-1})$
   (d) $\frac{1}{i\sqrt{2}}(1 + z^{-1})$

   Solution: (b). The given property $H_0(e^{-j\omega}) = \overline{H_0(e^{j\omega})}$ is satisfied by real sequences. Hence (b) is the only sequence for which the relation holds true.

3. $H_0(e^{j\omega})$ is a bandpass filter with cutoff frequencies at $\pi/4$ and $3\pi/4$. Thus $H_0(-e^{j\omega})$ is a ______ filter with cutoff frequency(s) at ______?

   (a) Bandpass, ($\pi/4$ and $3\pi/4$)
   (b) Bandstop, ($\pi/4$ and $3\pi/4$)
   (c) Highpass, $3\pi/4$
   (d) Highpass, $\pi/4$

Solution: (a) as $H_0(-e^{j\omega}) = H_0(e^{j(\omega+\pi)})$ and thus the frequency shifts by π making the bandpass filter as bandstop.

4. To derive the second member of the Daubechy filterbank family, we wrote the following condition:

$$(-1)^D H_0(z)H_0(z^-1) - H_0(-z)H_0(-z^-1) = c_0$$

What should D be?

(a) 1
(b) 2
(c) 3
(d) 4

Solution: (c). D for nth member of Daubechy wavelet family will be 2n - 1.

5. For the third member of the family, we'll get D = ?

(a) 2
(b) 3
(c) 4
(d) 5

Solution: (d) D for nth member of Daubechy wavelet family will be 2n - 1.

6. Let h(n) = 1, -1/2, 1/4, -1/8,... . Its z transform is represented by $H_0(z)$. Then, the constant term in $H_0(z)H_0(z^{-1})$ is

(a) 1
(b) 2
(c) 4/3
(d) ∞

Solution: (c). $H(z)H(z^-1) = Z(h[n] * h[-n])$. Thus the constant term of polynomial is given by $\sum h^2[n] = 1 + 1/4 + 1/16 + ... = \frac{4}{3}$

7. Which of the following transfer functions have the same frequency magnitude response?

(a) $1 + z^{-1}$,$1 - z^{-1}$
(b) $1 + z^{-1}$,z^{-1}
(c) $1 + z^{-1}$,$z^{-3} + z^{-4}$
(d) $z^{-1} + z^{-2}$,$1 - z^{-1}$

Solution: (c). $1 + z^{-1} = z^3(z^{-3} + z^{-4})$Time delay only changes phase part of the frequency response.

8. If $f(z) + f(-z) = g(z)$ where $f(.)$ is a polynomial function, then $g(.)$

   (a) is a constant.

   (b) has only non zero even powers

   (c) has only non zero odd powers

   (d) must have no constant term

   Solution: b. $f(z) + f(-z)$ contains the even powers only as the odd powers are cancelled out by each other.

9. Let $K(z) + K(-z) = z^{-4}$ and $K(z) = H_0(z)H_0(z^{-1})$. Then $\sum h[n]h[n-2] =$?

   (a) Information insufficient

   (b) 0

   (c) 1/2

   (d) $\sum h^2[n]$

   Solution (b). We know from given condition that K(z) has no power of z^{-2}. This implies that $\sum h[n]h[n-2]$ = coefficient of z^{-2} in $K(z)$ is 0.

10. Second member of the Daubechy filterbank family has:

    (a) 2 roots at z = -1

    (b) 2 roots at z = +1

    (c) 3 roots at z = -1

    (d) 3 roots at z = +1

    Solution: (a). As an extension from the Haar wavelet we impose an extra root at z = - 1 to find the second member of the family.

11. If $x[n] = [1, -1/2]$. Thus, let $Y(z) = log(X(z))$.
    Find the corresponding sequence $y[n]$. (Hint: $log(1-x) = -\sum_{k=1}^{\infty} \frac{x^k}{k}$ for $|x| < 1$).
    Note: The first element in the sequence is the zeroth element

    (a) $[0, \frac{1}{2}, \frac{-1}{4\times2}, \frac{1}{8\times3}...]$ for $|z| > \frac{1}{2}$

    (b) $[0, \frac{-1}{2}, \frac{-1}{4\times2}, \frac{-1}{8\times3}...]$ for $|z| > \frac{1}{2}$

    (c) $[0, \frac{-1}{2}, \frac{-1}{4\times2}, \frac{-1}{8\times3}...]$ for $|z| < \frac{1}{2}$

    (d) $[0, \frac{1}{2}, \frac{-1}{4\times2}, \frac{1}{8\times3}...]$ for $|z| < \frac{1}{2}$

    Solution: (b). $Y(z) = log(1 - \frac{1}{2}z^{-1}) = -\sum \frac{(\frac{1}{2^k}z^{-k})}{k}$ for $|\frac{1}{2}z^{-1}| < 1$. Thus the corresponding sequence of $Y(z)$ is (b).

12. If the third member of the Daubechy filterbank wavelet family is given by $h = [h_0, h_1, h_2, h_3, ..., h_n, ...]$, then which of the following statements are true?

    (a) $h_n = 0$ for n ¿ 6

(b) $h_n = 0$ for n ¿ 6 and n ¡ 3

(c) $h_n = 0$ for n ¿ 5

(d) $h_n = 0$ for n ¿ 5 and n ¡ 2

Solution (c). The length of the third member of the wavelet family is 6, hence $h_n = 0$ for n ¿ 5.

13. Second member of the Daubechy filterbank family annhilates ______

    (a) polynomials of degree 1 in the low pass filter.

    (b) polynomials of degree 2 in the low pass filter.

    (c) polynomials of degree 1 in the high pass filter.

    (d) polynomials of degree 2 in the high pass filter.

    Solution: (c). Haar wavelet annihilates a zero degree polynomial in the high pass filter and hence the second member is designed to annihilate degree 1 polynomials and hence is a 'stronger' high pass filter.

14. Which of the following is a minimum phase system?

    (a) $1 + 0.99z^{-1}$

    (b) $0.99 + z^{-1}$

    (c) $(1 + 0.99z^{-1})(1 + 2z^{-1})$

    (d) $(1 + z^{-1})(1 + 2z^{-1})$

    Solution: (a). Minimum Phase system is such that its inverse is causal and stable. Thus the poles and zeros of the system lie inside the unit circle.

15. Consider $h_0(t)$ to be a signal of support L. Define $h_n(t) := h_0(2^n t)$. Thus the signal $y(t) = h_0(t) * h_1(t) * h_2(t)$... has a length equal to

    (a) 3L/2

    (b) 5L/4

    (c) 7L/4

    (d) 2L

    Solution: (d). Convolution of two sequences leads to output of length equal to sum of each sequence. Thus total length of $y(t)$ = L + L/2 + L/4 + ... = 2L