Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis

Week 5 Assignment

April 3, 2017

1. \( G_0(z) \) and \( G_1(z) \) are low-pass and high pass synthesis side filters respectively and \( H_0(z) \) and \( H_1(z) \) are low-pass and high-pass analysis side filters respectively. Identify the correct relations between them along with the correct property.

   (a) \( G_0(z) = H_1(-z) \), \( G_1(z) = -H_0(-z) \), Alias Cancellation
   (b) \( G_0(z) = H_1(-z) \), \( G_1(z) = -H_0(-z) \), Power Complementarity
   (c) \( G_0(z) = H_1(z) \), \( G_1(z) = -H_0(-z) \), Alias Cancellation
   (d) \( G_0(z) = H_1(z) \), \( G_1(z) = -H_0(-z) \), Power Complementarity

   **Solution:** (a). Due to alias cancellation we require the following relation to hold true
   \[
   T_1(z) = G_0(z)H_0(-z) + G_1(z)H_1(-z) = 0.
   \]
   Hence if \( G_0(z) = H_1(-z) \), then we get the relation \( G_1(z) = -H_0(-z) \)

2. Which of the following Z-Transforms satisfy the property: \( H_0(e^{-j\omega}) = \overline{H_0(e^{j\omega})} \)

   (a) \( i + z^{-1} \)
   (b) \( 1 + z^{-1} \)
   (c) \( \frac{1}{\sqrt{2}}(i + z^{-1}) \)
   (d) \( \frac{1}{\sqrt{2}}(1 + z^{-1}) \)

   **Solution:** (b). The given property \( H_0(e^{-j\omega}) = \overline{H_0(e^{j\omega})} \) is satisfied by real sequences. Hence (b) is the only sequence for which the relation holds true.

3. \( H_0(e^{j\omega}) \) is a bandpass filter with cutoff frequencies at \( \pi/4 \) and \( 3\pi/4 \). Thus \( H_0(-e^{j\omega}) \) is a _____ filter with cutoff frequency(s) at _____?

   (a) Bandpass, \( \pi/4 \) and \( 3\pi/4 \)
   (b) Bandstop, \( \pi/4 \) and \( 3\pi/4 \)
   (c) Highpass, \( 3\pi/4 \)
   (d) Highpass, \( \pi/4 \)
Solution: (a) as \( H_0(-e^{j\omega}) = H_0(e^{j(\omega+\pi)}) \) and thus the frequency shifts by \( \pi \) making the bandpass filter as bandstop.

4. To derive the second member of the Daubech filterbank family, we wrote the following condition:

\[
(-1)^D H_0(z)H_0(z^{-1}) - H_0(-z)H_0(-z^{-1}) = c_0
\]

What should \( D \) be?
(a) 1
(b) 2
(c) 3
(d) 4

Solution: (c). \( D \) for nth member of Daubech wavelet family will be \( 2n - 1 \).

5. For the third member of the family, we’ll get \( D = ? \)
(a) 2
(b) 3
(c) 4
(d) 5

Solution: (d) \( D \) for nth member of Daubech wavelet family will be \( 2n - 1 \).

6. Let \( h(n) = 1, -1/2, 1/4, -1/8, \ldots \) . Its z transform is represented by \( H_0(z) \). Then, the constant term in \( H_0(z)H_0(z^{-1}) \) is
(a) 1
(b) 2
(c) 4/3
(d) \( \infty \)

Solution: (c). \( H(z)H(z^{-1}) = Z(h[n] * h[\neg n]) \). Thus the constant term of polynomial is given by \( \sum h^2[n] = 1 + 1/4 + 1/16 + \ldots = \frac{4}{3} \)

7. Which of the following transfer functions have the same frequency magnitude response?
(a) \( 1 + z^{-1}, 1 - z^{-1} \)
(b) \( 1 + z^{-1}, z^{-1} \)
(c) \( 1 + z^{-1}, z^{-3} + z^{-4} \)
(d) \( z^{-1} + z^{-2}, 1 - z^{-1} \)

Solution: (c). \( 1 + z^{-1} = z^3(z^{-3} + z^{-4}) \) Time delay only changes phase part of the frequency response.
8. If \( f(z) + f(-z) = g(z) \) where \( f(.) \) is a polynomial function, then \( g(.) \)
   (a) is a constant.
   (b) has only non zero even powers
   (c) has only non zero odd powers
   (d) must have no constant term

**Solution:** b. \( f(z) + f(-z) \) contains the even powers only as the odd
powers are cancelled out by each other.

9. Let \( K(z) + K(-z) = z^{-4} \) and \( K(z) = H_0(z)H_0(z^{-1}) \). Then \( \sum h[n]h[n - 2] = ? \)
   (a) Information insufficient
   (b) 0
   (c) 1/2
   (d) \( \sum h^2[n] \)

**Solution** (b). We know from given condition that \( K(z) \) has no power of
\( z^{-2} \). This implies that \( \sum h[n]h[n - 2] = \) coefficient of \( z^{-2} \) in \( K(z) \) is 0.

10. Second member of the Daubechy filterbank family has:
   (a) 2 roots at \( z = -1 \)
   (b) 2 roots at \( z = +1 \)
   (c) 3 roots at \( z = -1 \)
   (d) 3 roots at \( z = +1 \)

**Solution:** (a). As an extension from the Haar wavelet we impose an extra
root at \( z = -1 \) to find the second member of the family.

11. If \( x[n] = [1, -1/2] \). Thus, let \( Y(z) = log(X(z)) \).
    Find the corresponding sequence \( y[n] \). (Hint: \( log(1 - x) = - \sum_{k=1}^{\infty} \frac{x^k}{k} \) for
\( |x| < 1 \).

**Note:** The first element in the sequence is the zeroth element
   (a) \([0, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots] \) for \( |z| > \frac{1}{2} \)
   (b) \([0, -\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots] \) for \( |z| > \frac{1}{2} \)
   (c) \([0, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots] \) for \( |z| < \frac{1}{2} \)
   (d) \([0, \frac{1}{2}, -\frac{1}{4}, -\frac{1}{8}, \ldots] \) for \( |z| < \frac{1}{2} \)

**Solution:** (b). \( Y(z) = log(1 - \frac{1}{2}z^{-1}) = - \sum_{k=1}^{\infty} \frac{(\frac{1}{2}z^{-1})^k}{k} \) for \( |\frac{1}{2}z^{-1}| < 1 \). Thus
the corresponding sequence of \( Y(z) \) is (b).

12. If the third member of the Daubechy filterbank wavelet family is given by
    \( h = [h_0, h_1, h_2, h_3, ..., h_n, ...] \), then which of the following statements are
true?
   (a) \( h_n = 0 \) for \( n \neq 6 \)
(b) \( h_n = 0 \) for \( n \geq 6 \) and \( n \geq 3 \)
(c) \( h_n = 0 \) for \( n \geq 5 \)
(d) \( h_n = 0 \) for \( n \geq 5 \) and \( n \geq 2 \)

\textbf{Solution} (c). The length of the third member of the wavelet family is 6, hence \( h_n = 0 \) for \( n \geq 5 \).

13. Second member of the Daubechy filterbank family annihilates ______

(a) polynomials of degree 1 in the low pass filter.
(b) polynomials of degree 2 in the low pass filter.
(c) polynomials of degree 1 in the high pass filter.
(d) polynomials of degree 2 in the high pass filter.

\textbf{Solution}: (c). Haar wavelet annihilates a zero degree polynomial in the high pass filter and hence the second member is designed to annihilate degree 1 polynomials and hence is a 'stronger' high pass filter.

14. Which of the following is a minimum phase system?

(a) \( 1 + 0.99z^{-1} \)
(b) \( 0.99 + z^{-1} \)
(c) \( (1 + 0.99z^{-1})(1 + 2z^{-1}) \)
(d) \( (1 + z^{-1})(1 + 2z^{-1}) \)

\textbf{Solution}: (a). Minimum Phase system is such that its inverse is causal and stable. Thus the poles and zeros of the system lie inside the unit circle.

15. Consider \( h_0(t) \) to be a signal of support \( L \). Define \( h_n(t) := h_0(2^n t) \). Thus the signal \( y(t) = h_0(t) * h_1(t) * h_2(t) \ldots \) has a length equal to

(a) \( 3L/2 \)
(b) \( 5L/4 \)
(c) \( 7L/4 \)
(d) \( 2L \)

\textbf{Solution}: (d). Convolution of two sequences leads to output of length equal to sum of each sequence. Thus total length of \( y(t) = L + L/2 + L/4 + \ldots = 2L \).