

## Course outline

How does an NPTEL online course work?

## Propositional Logic

## Predicate Logic, Proof Strategies and Induction

## Sets and Relations

## Equivalence Relations, Partitions, Partial Orderings and Functions

- Equivalence Relation
- Equivalence Relations and Partitions
- Partial Ordering
- Functions
- Tutorial 4: Part I
- Tutorial 4: Part II
- Quiz : Week 4 Assignment
- Tutorial Problem

## Theory of Countability

## Combinatorics Part I

## Combinatorics Part II

## Graph Theory Part I

## Graph Theory Part II

## Number theory

## Abstract Algebra : Part I

## Abstract Algebra : Part II

## Video download

## Live Session

## Text transcripts

# Week 4 Assignment

The due date for submitting this assignment has passed.

**Due on 2021-02-17, 23:59 IST.**

As per our records you have not submitted this assignment.

Equivalence relation, partitions, equivalence classes, poset, Hasse diagram, topological sort, functions

1) Select the incorrect option(s)

**2 points**

- The relation  $R$  on  $\mathbb{Z}$ ,  $aRb$  if  $a^2 - b^2 \leq 3$ , is an equivalence relation
- The relation  $R$  on  $\mathbb{Z}$ ,  $aRb$  if  $a \neq b$ , is an equivalence relation
- The relation  $R$  on  $\mathbb{Z}$ ,  $aRb$  if  $(a + b) \equiv 0 \pmod{10}$ , is an equivalence relation

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**

The relation  $R$  on  $\mathbb{Z}$ ,  $aRb$  if  $a^2 - b^2 \leq 3$ , is an equivalence relation  
The relation  $R$  on  $\mathbb{Z}$ ,  $aRb$  if  $a \neq b$ , is an equivalence relation  
The relation  $R$  on  $\mathbb{Z}$ ,  $aRb$  if  $(a + b) \equiv 0 \pmod{10}$ , is an equivalence relation

2) Which of the following is/are true?

**1 point**

- The inverse of a function  $f$  exists if  $f$  is injective only
- The inverse of a function  $f$  exists if  $f$  is surjective only
- The inverse of a function  $f$  is defined if  $f$  is both injective and surjective
- A function that is neither injective or surjective is sometimes invertible

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**

The inverse of a function  $f$  is defined if  $f$  is both injective and surjective

3) Select the incorrect option(s) from the following:

**1 point**

- A relation  $R$  on a set  $A$  is called partial ordering if it is reflexive, anti-symmetric and transitive.
- A Hasse diagram will contain at most one maximal element and at most one minimal element
- In a given relation  $R$  which is reflexive, symmetric and antisymmetric, if some elements cannot be compared with other elements, then  $R$  will constitute a poset
- Define  $f: \mathbb{Z} \rightarrow \mathbb{Z}$ ,  $f(x) = x^2 + x$ ; then  $f$  is a one-to-one function.

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**

A Hasse diagram will contain at most one maximal element and at most one minimal element

Define  $f: \mathbb{Z} \rightarrow \mathbb{Z}$ ,  $f(x) = x^2 + x$ ; then  $f$  is a one-to-one function.

 4) The number of distinct equivalence classes based on the relation  $R = \{(a, b) : a \equiv b \pmod{p}\}$  where  $p$  is a prime integer :

**1 point**

- 1
- $(p-1)/2$
- $p-1$
- $p$

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**

$p$

 5) Consider the poset on the relation  $R = \{(a, b) : a \text{ divides } b\}$  over the set of positive integers excluding one ( $\mathbb{Z}^+ - 1$ ). The minimal element(s) of the given poset is (are):

**1 point**

- 2 is the minimal element
- The set of elements,  $\{2\} \cup \{3, 5, 7, 9, \dots\}$
- The set of odd numbers  $3, 5, 7, 9, \dots$
- None of the given options

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**

None of the given options

 6) Consider the functions  $f_1: A \rightarrow B$  and  $f_2: A \rightarrow B$  such that:

**1 point**

$$f_1(x) = 2x + 4,$$

$$f_2(x) = 4x + 2.$$

where  $A$  is set of integers and  $B$  is set of even integers. Which of the following are true about the function  $f$ ?

- Both  $f_1$  and  $f_2$  are one-one
- Both  $f_1$  and  $f_2$  are onto
- Both  $f_1$  and  $f_2$  are bijections
- Only  $f_1$  is a bijection
- $f_2$  is not a bijection

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**

Both  $f_1$  and  $f_2$  are one-one

Only  $f_1$  is a bijection

$f_2$  is not a bijection

 7) Let  $\mathcal{F}$  be the set of all functions  $f: \mathbb{N}_0 \rightarrow \mathbb{R}^+$  where  $\mathbb{N}_0$  is the set of all whole numbers i.e.,  $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ . Let  $\Theta$  be a relation on  $\mathcal{F}$  such that  $(f, g) \in \Theta$  for any two functions  $f, g \in \mathcal{F}$  if and only if there exists positive constants  $c_1, c_2$  and  $n_0$  such that  $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$  for all  $n \geq n_0$ . Choose the correct statement(s).

**1 point**

- $\Theta$  is reflexive
- $\Theta$  is transitive
- $\Theta$  is symmetric
- $\Theta$  is an equivalence relation

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**

$\Theta$  is reflexive

$\Theta$  is transitive

$\Theta$  is symmetric

$\Theta$  is an equivalence relation

 8) Let  $\mathcal{F}$  be the set of all functions  $f: \mathbb{N}_0 \rightarrow \mathbb{R}^+$  where  $\mathbb{N}_0$  is the set of all whole numbers i.e.,  $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ . Let  $O$  be a relation on  $\mathcal{F}$  such that  $(f, g) \in O$  for any two functions  $f, g \in \mathcal{F}$  if and only if there exists positive constants  $c$  and  $n_0$  such that  $0 < f(n) \leq c g(n)$  for all  $n \geq n_0$ . Choose the correct statement(s).

**1 point**

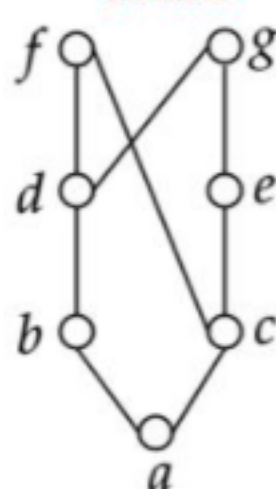
- $O$  is a partial order
- $O$  is a total order
- $O$  is an equivalence relation
- None of the given options

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**

None of the given options

9) Choose the correct statement(s) based on the Hasse diagram given below

**1 point**


- $g$  is the greatest element
- $a$  is the least element
- $(a, b, d, c, f, e, g)$  is a valid topological ordering
- $(a, b, c, d, e, f, g)$  is a valid topological ordering

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**

$a$  is the least element

$(a, b, d, c, f, e, g)$  is a valid topological ordering

$(a, b, c, d, e, f, g)$  is a valid topological ordering