Assignment 0

Due on 2020-02-25, 23:59 HKT.

1. If $P(U, V) = 0.5$, $P(U = v) = 0.7$ and $P(U = v | X) = 0.6$. What is the range of values of $P(U, V) | X$?  
   \[ \text{Answer: } 0 \leq P(U, V) | X \leq 0.7 \]

2. Find the variance of the random variable $X$, where $X$ follows the distribution $f(x) \propto (x + 1) + 0.7 (x + 1)^2$.
   \[ \text{Answer: } \frac{18}{7} \]

3. If $X$ and $Y$ are independent random variables, which of the following is/are true?
   \[ \begin{align*}
   & \text{a) } E(X) = E(Y) \\
   & \text{b) } E(X + Y) = E(X) + E(Y) \\
   & \text{c) } E(X + Y) = E(X) + E(Y) \\
   & \text{d) } \text{The above answers are all true.}
   \end{align*} \]

4. Which of the following is/are true for a vector norm function?  
   \[ \begin{align*}
   & \text{a) } \|X\| = \|X\|_2 \\
   & \text{b) } \|X\|_1 = \|X\|_1 \\
   & \text{c) } \|X\|_2 = \|X\|_2 \\
   & \text{d) } \|X\|_\infty = \|X\|_\infty \\
   & \text{e) } \text{The above answers are all true.}
   \end{align*} \]

5. Which of the following is/are true for a matrix $A$?
   \[ \begin{align*}
   & \text{a) } \det(A) = \det(A^T) \\
   & \text{b) } \det(A) > 0 \\
   & \text{c) } \det(A) < 0 \\
   & \text{d) } \det(A) = 0 \\
   & \text{e) } \text{The above answers are all true.}
   \end{align*} \]

6. For a matrix $A$, its transpose $A^T$ is a square matrix with eigenvalues of $A$. If $X$ is an orthogonal matrix, what is the transpose of $X$?
   \[ \text{Answer: } X^T \]

7. Which of the following is/are true for a vector $v$?
   \[ \begin{align*}
   & \text{a) } v \cdot v = \|v\|^2 \\
   & \text{b) } v \cdot v = \|v\|^2 \\
   & \text{c) } v \cdot v = \|v\|^2 \\
   & \text{d) } \text{The above answers are all true.}
   \end{align*} \]

8. For a vector $v$, its transpose $v^T$ is a square matrix with eigenvalues of $v$. If $A$ is an orthogonal matrix, what is the norm of $A^T$?
   \[ \text{Answer: } 1 \]

9. Which of the following is/are true for a matrix $A$?
   \[ \begin{align*}
   & \text{a) } \text{The eigenvalues of } A \text{ are real.} \\
   & \text{b) } \text{The eigenvalues of } A \text{ are all positive.} \\
   & \text{c) } \text{The eigenvalues of } A \text{ are all negative.} \\
   & \text{d) } \text{The above answers are all true.}
   \end{align*} \]

10. Which of the following is/are true for a matrix $A$?
    \[ \begin{align*}
    & \text{a) } A \text{ is a symmetric matrix.} \\
    & \text{b) } A \text{ is an orthogonal matrix.} \\
    & \text{c) } A \text{ is a diagonal matrix.} \\
    & \text{d) } \text{The above answers are all true.}
    \end{align*} \]