Assignment 9

1. Consider two connected graphs (a) and (b), with the same number of vertices. Determine if there exists a path connecting vertices (A) and (D) in (b) but not in (a).

2. Given a connected graph, prove that if an edge is removed, the graph becomes disconnected only if the edge was a bridge.

3. Prove that if G and H are connected graphs, then the graph G + H is also connected. (Hint: Use the definition of connectedness and show that any two vertices in the new graph can be connected through a path involving at least one new edge.)

4. Let G be a graph and let x be a vertex of degree 1. Show that G is connected if and only if there exists a path in G that includes x.

5. A graph is Eulerian if it contains an Eulerian cycle, a cycle that visits every edge exactly once. Prove that a connected graph is Eulerian if and only if every vertex has even degree.

6. Prove that if G is a connected graph and e is an edge of G, then the graph G - e (with e removed) is connected if and only if the removal of e does not disconnect G.

7. Consider two graphs G and H. Prove that if G is a subgraph of H, then G is connected if G contains a path connecting any two of its vertices, and H contains a path connecting any two of its vertices, then H is connected.

8. Let G be a connected graph with n vertices and let d be the minimum degree of a vertex in G. Prove that the maximum degree of a vertex in G is at most 2n - 1 - d.

9. Formulate and prove a theorem that describes the relationship between the number of vertices, edges, and the maximum degree in a simple connected graph.