NPTEL » Design and analysis of algorithms (course)

Announcements (announcements)  About the Course

Ask a Question (forum)  Progress (student/home)  Mentor (student/mentor)

Unit 22 - Week 7 Quiz

Week 7 Quiz

The due date for submitting this assignment has passed. Due on 2020-03-18, 23:59 IST.

As per our records you have not submitted this assignment.

All questions carry equal weightage. You may submit as many times as you like within the deadline. Your final submission will be graded.

Your final exams are over and you are catching up on sports on TV. You have a schedule of interesting matches from all over the world during the next week. You hate to start or stop watching a match midway, so your aim is to watch as many complete matches as possible during the week.

Suppose there are n such matches \( \{M_1, M_2, ..., M_n\} \) available during the coming week. The matches are ordered by starting time, so for each \( i \in \{1, 2, ..., n-1\} \), \( M_i \) starts before \( M_{i+1} \). However, match \( M_i \) may not end before \( M_{i+1} \) starts, so for each \( i \in \{1, 2, ..., n-1\} \), Next\([i]\) is the smallest \( j > i \) such that \( M_j \) starts after \( M_i \) finishes.

Given the sequence \( \{M_1, M_2, ..., M_n\} \) and the values Next\([i]\) for each \( i \in \{1, 2, ..., n-1\} \), your aim is to compute the maximum number of complete matches that can be watched.

1) Let Watch\([i]\) denote the maximum number of complete matches that can be watched among \( \{M_i, M_{i+1}, ..., M_n\} \). Which of the following is a correct recursive formulation of Watch\([i]\)?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Watch([1]) = 1</td>
<td></td>
</tr>
<tr>
<td>Watch([i]) = max(Watch([i-1]) + 1, Watch([\text{Next}[i-1])])</td>
<td>( i \in {2, 3, ..., n} )</td>
</tr>
<tr>
<td>Watch([n]) = 1</td>
<td></td>
</tr>
<tr>
<td>Watch([i]) = max(1 + Watch([\text{Next}[i]]), Watch([i+1]])</td>
<td>( i \in {1, 2, ..., n-1} )</td>
</tr>
<tr>
<td>Watch([1]) = 1</td>
<td></td>
</tr>
<tr>
<td>Watch([i]) = max(Watch([i-1]), 1 + Watch([\text{Next}[i-1])])</td>
<td>( i \in {2, 3, ..., n} )</td>
</tr>
</tbody>
</table>
2) What is the size of the memo table for this problem?  

- $n^2$  
- $n+1$  
- $n$  
- $n-1$  

No, the answer is incorrect.  
Score: 0  
Feedback:  
Either watch match $i$ and continue from Next[$i$] or skip match $i$ and continue from match $i+1$.

Accepted Answers:  
Watch[$n$] = 1  
Watch[$i$] = max(1 + Watch[Next[$i$]], Watch[$i$ + 1]), $i \in \{1, 2, ..., n-1\}$

3) What is a good order to compute Watch[$i$] using dynamic programming?  

- From Watch[$n$] to Watch[$1$]  
- From Watch[$1$] to Watch[$n$]  
- Either from Watch[$1$] to Watch[$n$] or from Watch[$n$] to Watch[$1$]  
- None of these  

No, the answer is incorrect.  
Score: 0  
Feedback:  
The base case is Watch[$n$], so start with Watch[$n$] and work backwards to Watch[$1$].

Accepted Answers:  
From Watch[$n$] to Watch[$1$]

4) How much time will it take to compute Watch[$1$] using dynamic programming?  

- $O(n^3)$  
- $O(n^2)$  
- $O(n \log n)$  
- $O(n)$  

No, the answer is incorrect.  
Score: 0  
Feedback:  
The table is of size $n$ and can be filled in a single pass. Each entry Watch[$i$] requires checking two values, Watch[$i+1$] and Watch[Next[$i$]], so the time taken is $O(n)$.

Accepted Answers:  
$O(n)$

5) Suppose the list of matches to be watched is presented in the form

- $n$
where each match $M_i$ is represented by a pair $(S_i, T_i)$ indicating its starting time and ending time.
To be able to watch both $M_i$ and $M_j$, for $j > i$, it must be the case that $S_j > T_i$.

What is the maximum number of matches you can watch in this case?

- 10
- 9
- 8
- 7

No, the answer is incorrect.
Score: 0
Feedback:
*Evaluate the recurrence. The values of Watch[i] as a list are*

$$[8, 8, 7, 6, 6, 6, 5, 4, 3, 2, 2, 1]$$

**Accepted Answers:**
8