Assignment 6

In this project, you will be exploring the properties of the function f(x, y) described in the following equation:

f(x, y) = x^2 + y^2

1. Describe the function's behavior when x = 0 and y = 0. What happens to the function as x or y becomes very large?
   - When x = 0 and y = 0, f(x, y) = 0.
   - As x or y becomes very large, f(x, y) increases without bound.

2. Identify the critical points of the function and classify them as local maxima, local minima, or saddle points.
   - The function has one critical point at (0, 0).
   - This point is a local minimum.

3. Determine the domain and range of the function.
   - Domain: x, y ∈ ℝ
   - Range: f(x, y) ≥ 0

4. Sketch the graph of the function over the interval -5 ≤ x ≤ 5 and -5 ≤ y ≤ 5.
   - The graph is a paraboloid.

5. Find the level curves of the function and determine their asymptotic behavior.
   - Level curves are circles centered at the origin.
   - As the level increases, the circles grow larger.

6. Calculate the partial derivatives of the function with respect to x and y.
   - ∂f/∂x = 2x
   - ∂f/∂y = 2y

7. Evaluate the double integral of the function over the region R defined by -1 ≤ x ≤ 1 and -1 ≤ y ≤ 1.
   - ∫∫R f(x, y) dA = ∫[-1 to 1] ∫[-1 to 1] (x^2 + y^2) dx dy
   - The result is 4/3.

8. Determine the gradient of the function at the point (1, 1).
   - Gradient = (2, 2)

9. Find the equation of the tangent plane to the surface at the point (1, 1, 2).
   - Tangent plane: z = 2 + 2(x - 1) + 2(y - 1)

10. Calculate the directional derivative of the function in the direction of the vector v = (1, 1).
    - Directional derivative = ∇f · v = 4

11. Find the critical points of the function subject to the constraint x^2 + y^2 = 1.
    - The function is constant on the constraint.

12. Use the method of Lagrange multipliers to find the absolute maximum and minimum values of the function on the unit circle.
    - Absolute maximum: 2
    - Absolute minimum: 0