Assignment 8

1. Given the following system of equations, find the values of $x$ and $y$.

$$
\begin{align*}
3x + 2y &= 14 \\
4x - y &= 5
\end{align*}
$$

2. Consider the function $f(x) = x^2 - 4x + 4$. Sketch the graph of $f(x)$ and determine the roots of the function.

3. Find the equation of the line that passes through the points $(1, 2)$ and $(3, 8)$.

4. Solve the system of linear equations:

$$
\begin{align*}
x + 2y &= 7 \\
2x - y &= 1
\end{align*}
$$

5. Determine if the vectors $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$ are linearly independent.

6. Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$.

7. A cylindrical tank has a radius of 5 meters and a height of 10 meters. The tank is filled with water to a depth of 8 meters. Calculate the volume of water in the tank.

8. A company produces two types of products, A and B. The profit from producing A is $5 per unit, and the profit from producing B is $7 per unit. The company has a total of 100 units of labor and 150 units of materials available. Each unit of A requires 2 units of labor and 3 units of materials, while each unit of B requires 4 units of labor and 2 units of materials. How many units of A and B should the company produce to maximize profit?