Assignment 6

The due date for submitting this assignment has passed. Due on 2019-03-13, 23:59 IST.
As per our records you have not submitted this assignment.

1) Find the first six terms of the sequence \( \{a_n\} \) defined by the recurrence relation \( a_n = a_{n-1} + a_{n-2} \) for \( n \geq 3 \), with the initial conditions \( a_0 = 1 \), \( a_1 = 2 \), \( a_2 = 0 \).
   
   a. 1, 3, 4, 7, 11
   b. 1, 3, 4, 7, 10
   c. 1, 2, 0, 1, 3, 3
   d. 1, 2, 0, 1, 3, 4

No, the answer is incorrect.
Score: 0
Accepted Answers:
   c.

2) A vending machine dispensing water accepts only \( \text{₹} \) 1 coins, \( \text{₹} \) 2 coins, and \( \text{₹} \) 2 tokens. Fill recurrence relation for the number of ways to deposit \( \text{₹} n \) in the vending machine, where order in which the coins and tokens are deposited matters. (Let \( a_n \) be the number of way deposit \( \text{₹} n \) in the vending machine.)

   a. \( a_n = a_{n-1} + a_{n-2} \) for \( n \geq 3 \), with the initial conditions \( a_1 = 1 \) and \( a_2 = 3 \).
   b. \( a_n = a_{n-1} \) for \( n \geq 2 \), with the initial conditions \( a_1 = 1 \).
   c. \( a_n = 2a_{n-2} \) for \( n \geq 3 \), with the initial conditions \( a_1 = 1 \) and \( a_2 = 3 \).
   d. \( a_n = a_{n-1} + 2a_{n-2} \) for \( n \geq 3 \), with the initial conditions \( a_1 = 1 \) and \( a_2 = 3 \).

a. 
b. 
c. 
d.
Find a recurrence relation for the number of bit strings of length $n$ that contain a pair of consecutive 1's. (Let $a_n$ be the number of bit strings of length $n$ containing a pair of consecutive 1's.)

\[ a_n = a_{n-1} + 2a_{n-2} \text{ for } n \geq 3, \text{ with the initial conditions } a_1 = 0 \text{ and } a_2 = 1. \]

\[ a_n = a_{n-1} + a_{n-2} + 2^{n-2} \text{ for } n \geq 3, \text{ with the initial conditions } a_1 = 0 \text{ and } a_2 = 1. \]

\[ a_n = 2a_{n-1} + 2a_{n-2} \text{ for } n \geq 3, \text{ with the initial conditions } a_1 = 0 \text{ and } a_2 = 1. \]

\[ a_n = 2a_{n-1} + 4a_{n-2} \text{ for } n \geq 3, \text{ with the initial conditions } a_1 = 0 \text{ and } a_2 = 1. \]

No, the answer is incorrect.
Score: 0
Accepted Answers:

4)
Find a recurrence relation for the number of ternary strings (A string that contains only 0s and 2s) of length $n$ that do not contain two consecutive 0s. (Let $a_n$ be the number of ternary strings of length $n$ that do not contain two consecutive 0's.)

\[ a_n = a_{n-1} + a_{n-2} \text{ for } n \geq 2, \text{ with the initial conditions } a_0 = 1, a_1 = 3. \]

\[ a_n = a_{n-1} + 2a_{n-2} \text{ for } n \geq 2, \text{ with the initial conditions } a_0 = 1, a_1 = 3. \]

\[ a_n = 2a_{n-1} + 8a_{n-2} \text{ for } n \geq 2, \text{ with the initial conditions } a_0 = 1, a_1 = 3. \]

\[ a_n = 2a_{n-1} + 2a_{n-2} \text{ for } n \geq 2, \text{ with the initial conditions } a_0 = 1, a_1 = 3. \]

No, the answer is incorrect.
Score: 0
Accepted Answers:

5)
Find a recurrence relation for the number of ways to climb $n$ stairs if the person climbing stairs can take one stair or two stairs at a time. (Let $a_n$ be the number of ways to climb $n$ stairs)

\[ a_n = a_{n-1} + 2a_{n-2} \text{ for } n \geq 3, \text{ with the initial conditions } a_1 = 1 \text{ and } a_2 = 2. \]

\[ a_n = a_{n-1} \text{ for } n \geq 2, \text{ with the initial condition } a_1 = 1. \]

\[ a_n = a_{n-2} \text{ for } n \geq 3, \text{ with the initial conditions } a_1 = 1 \text{ and } a_2 = 2. \]

\[ a_n = a_{n-1} + a_{n-2} \text{ for } n \geq 3, \text{ with the initial conditions } a_1 = 1 \text{ and } a_2 = 2. \]

No, the answer is incorrect.
Score: 0
Accepted Answers:


6) Messages are transmitted over a communication channel using three signals. The transmission of these signals requires 1 μsec, 2 μsec, and 3 μsec. Find a recurrence relation for the number of different messages consisting of sequences of these three signals, where each signal in a message is immediately followed by the next signal, that can be sent in $n$ μsec. (Let $a_n$ be the number of ways to transmit messages over the communication channel.)

a. $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 4$, with the initial conditions $a_1 = 1$, $a_2 = 2$, and $a_3 = 4$.

b. $a_n = a_{n-1} + 2a_{n-2} + 3a_{n-3}$ for $n \geq 4$, with initial conditions $a_1 = 1$, $a_2 = 2$, and $a_3 = 4$.

c. $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$, with the initial conditions $a_1 = 1$ and $a_2 = 2$.

d. $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$, with the initial conditions $a_1 = 1$ and $a_2 = 2$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
a.
b.
c.
d.

7) There are $n$ lines drawn in a plane such that no two lines are parallel and no three lines are concurrent. If the plane is thereby divided into $a_n$ regions, find a recurrence relation for $(a_n)$.

a. $a_n = 2a_{n-1}$ for $n \geq 2$, $a_1 = 2$.

b. $a_n = a_{n-1} + n - 1$ for $n \geq 2$, $a_1 = 2$.

c. $a_n = a_{n-1} + n$ for $n \geq 2$, $a_1 = 2$.

d. None of the above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
c.

8) Find a recurrence relation for the number of bit strings that contain the string ‘01’. Let $a_n$ be the number of the bit strings of length $n$.

a. $a_n = 2a_{n-1} + n - 1$ for $n \geq 2$, $a_1 = 0$.

b. $a_n = 2a_{n-1}$ for $n \geq 2$, $a_1 = 0$.

c. $a_n = a_{n-1} + n - 1$ for $n \geq 2$, $a_1 = 0$.

d. None of the above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
9) Find a recurrence relation for the number of ternary strings that do not contain consecutive symbols that are the same. Let \( a_n \) be the number of the ternary strings of length \( n \).
   \[ \begin{align*}
   a. & \quad a_n = 2a_{n-1} \text{ for } n \geq 2, \quad a_1 = 3 \\
   b. & \quad a_n = a_{n-1} + a_{n-2} \text{ for } n \geq 3, \quad a_1 = 3, \quad a_2 = 6 \\
   c. & \quad a_n = a_{n-1} + n + 1 \text{ for } n \geq 2, \quad a_1 = 3 \\
   d. & \quad \text{None of the above}
   \end{align*} \]

   No, the answer is incorrect.
   Score: 0
   Accepted Answers:
   a.

10) Let \( a_n \) be the number of the \( n \)-letter strings that can be formed using the letters \( A \), \( B \), and \( C \) so that every \( A \) in the strings has to be immediately followed by a \( B \). Find the recurrence relation for \( \{a_n\} \).
   \[ \begin{align*}
   a. & \quad a_n = a_{n-1} + a_{n-2} \text{ for } n \geq 3, \quad a_1 = 2, \quad a_2 = 5 \\
   b. & \quad a_n = 2a_{n-1} \text{ for } n \geq 2, \quad a_1 = 2 \\
   c. & \quad a_n = 2a_{n-1} + 2a_{n-2} \text{ for } n \geq 3, \quad a_1 = 2, \quad a_2 = 5 \\
   d. & \quad a_n = 2a_{n-1} + a_{n-2} \text{ for } n \geq 3, \quad a_1 = 2, \quad a_2 = 5
   \end{align*} \]

   No, the answer is incorrect.
   Score: 0
   Accepted Answers:
   d.

11) A factory makes custom sports cars at an increasing rate. In the first month only one car is made, in the second month two cars are made, and so on, with \( n \) cars made in the \( n \)-th month. Find a recurrence relation for \( a_n \), the number of cars produced in the first \( n \) months by the factory.
   \[ \begin{align*}
   a. & \quad a_n = 3a_{n-1} + a_{n-2} \text{ for } n \geq 2, \quad a_0 = 0, \quad a_1 = 1 \\
   b. & \quad a_n = a_{n-1} + a_{n-2} \text{ for } n \geq 2, \quad a_0 = 0, \quad a_1 = 1 \\
   c. & \quad a_n = a_{n-1} + n \text{ for } n \geq 2, \quad a_1 = 1 \\
   d. & \quad \text{None of the above}
   \end{align*} \]

   No, the answer is incorrect.
   Score: 0
12) Find a recursive definition of the sequence \( \{a_n\} \), \( n = 1, 2, 3, \ldots \), if \( a_n = 2n^2 + 1 \).
   a. \( a_n = a_{n-1} + 6 \) for \( n \geq 2 \) with \( a_1 = 3 \)
   b. \( a_n = a_{n-1} + 4n - 3 \) for \( n \geq 2 \) with \( a_1 = 3 \)
   c. \( a_n = a_{n-1} + 2n - 2 \) for \( n \geq 2 \) with \( a_1 = 3 \)
   d. \( a_n = a_{n-1} + 4n - 2 \) for \( n \geq 2 \) with \( a_1 = 3 \)

No, the answer is incorrect.
Score: 0

Accepted Answers: 
- c.

13) Let the set \( S \) be defined by \( 1 \in S \), and \( s + r \in S \) whenever \( s \in S \) and \( r \in S \). The set \( S \) is
   a. the set of odd positive integers.
   b. the set of positive integers.
   c. the set of positive integers not divisible by 5.
   d. None of the above

No, the answer is incorrect.
Score: 0

Accepted Answers: 
- d.

14) Let the set \( A \) of bit strings be defined recursively by \( \lambda \in A \), and \( 0x1 \in A \) if \( x \in A \) where \( \lambda \) is the empty string. The set \( A \) contains
   a. all bit strings of even length.
   b. all bit strings where every 0 is immediately followed by 1.
   c. all bit strings consisting of \( n \) consecutive 0s followed by \( n \) consecutive 1s for nonnegative integer \( n \).
   d. all bit strings consisting of equal number of 0s and equal number of 1s.

No, the answer is incorrect.
Score: 0

Accepted Answers: 
- c.

15)
The two-argument Ackermann-Peter function is defined as follows for nonnegative integer and $n$:

$$A(m, n) = n + 1, \text{ if } m = 0$$

$$= A(m - 1, 1), \text{ if } m > 0 \text{ and } n = 0$$

$$= A(m - 1, A(m, n - 1)), \text{ if } m > 0 \text{ and } n > 0.$$

Find $A(1, 3)$ and $A(2, 1)$.

a. 4, 4
b. 5, 5
c. 5, 4
d. 4, 5

No, the answer is incorrect.

Score: 0

Accepted Answers:
b.