Assignment 7

Problem Set 28 - 20.051

1. Suppose $T$ is a transition relation. Prove or disprove the following statements.

a. If $T^{-1}$ is symmetric, then $T$ is reflexive.

b. If $T^{-1}$ is transitive, then $T$ is symmetric.

c. If $T$ is transitive, then $T^{-1}$ is transitive.

d. If $T$ is reflexive, then $T^{-1}$ is symmetric.

2. Consider the following digraph $G = (V, E)$.

![Graph Image]


c. Find the transitive closure of $G$.

3. Which of the following statements are true or false?

a. Every connected graph contains at least one cycle.

b. Every tree has exactly one cycle.

c. The transitive closure of a reflexive relation is reflexive.

4. Let $R$ be a relation on a set $A$. Define $R^*$ as the transitive closure of $R$. Suppose $R$ is symmetric. Prove that $R^*$ is also symmetric.

5. Let $A = \{1, 2, 3, 4\}$. Define the relation $R$ on $A$ by $R = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$. Find the transitive closure $R^*$ of $R$.

6. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3\}$. Define the relation $R: A \times B$ by $R = \{(1, 1), (1, 2), (2, 2), (3, 2), (4, 3)\}$. Find the transitive closure $R^*$ of $R$.