Assignment 8

The due date for submitting this assignment has passed.

Due on 2021-03-17, 23:59 IST.

As per our records you have not submitted this assignment.

1) There are n people at a party. They have all kept their hats in a dark closet. At the end of the party, they all go and randomly take a hat from the closet. Let p_n be the probability that no person will pick his/her own hat. What is $\lim_{n\to\infty} p_n$? *4 points*

- $\frac{1}{\pi}$
- $\frac{1}{\pi^2}$
- $\frac{1}{e}$
- $\frac{1}{e^2}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{1}{e}$

2) Let $\mathbb{F}$ be a finite field of order m (i.e., $\mathbb{F}$ has m elements). Define the family of functions *4 points*
$H = \{h_{(a_0,a_1\ldots,a_{k-1})} : \mathbb{F} \to \mathbb{F} \mid a_0, a_1, \ldots, a_{k-1} \in \mathbb{F}\}$
where $h_{(a_0,a_1\ldots,a_{k-1})}(x) = a_0 + a_1x + \ldots a_{k-1}x^{k-1}$. In other words, every tuple $(a_0, a_1 \ldots, a_{k-1}) \in \mathbb{F}^k$ defines a polynomial and H consists of all these polynomials.
For distinct $x_1, \ldots, x_k \in \mathbb{F}$ and for any $y_1, \ldots, y_k \in \mathbb{F}$, what is $Pr_{h\in H}[h(x_1) = y_1 \wedge \ldots h(x_k) = y_k]$?

- $m(m-2)/m^k$
- $1/m^{k-1}$
- $1/m^k$
- Cannot be determined

No, the answer is incorrect.
Score: 0

Accepted Answers:
$1/m^k$

3) Which of the following problems are $\#P$ complete? *2 points*

- SAT
- $\#SAT$
- $CLIQUE$
- $\#CLIQUE$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\#SAT$
$\#CLIQUE$

4) If the number of solutions to a SAT instance can be computed in polynomial time, then *4 points*

- $NP \neq coNP$
- $\Sigma_3^p = \Pi_3^p$
- $P = PSPACE$
- PH has a complete problem

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\Sigma_3^p = \Pi_3^p$
PH has a complete problem

5) A complexity class B is said to be 'low' for a complexity class A if $A^B = A$ (i.e., A with an oracle for B is equal to A). Which of the following classes are low for themselves? *2 points*

- P
- NP
- BPP
- $\oplus P$

No, the answer is incorrect.
Score: 0

Accepted Answers:
P
BPP
$\oplus P$

6) Which of the following is a consequence of the Valiant Vazirani theorem? *6 points*

- If $USAT$ has a polynomial time algorithm, then $BPP = RP$
- If $USAT$ has a polynomial time algorithm, then $NP = RP$
- $NP \subseteq BPP^{\oplus P}$
- $NP^{\oplus P} \subseteq BPP^{\oplus P}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $USAT$ has a polynomial time algorithm, then $NP = RP$
$NP \subseteq BPP^{\oplus P}$
$NP^{\oplus P} \subseteq BPP^{\oplus P}$

7) A language L is in the class PP iff there exists a probabilistic Turing machine M, such that $\begin{array}{l} x \in L \implies Pr[M(x)=1] > 1/2 \\ x \notin L \implies Pr[M(x)=1] \leq 1/2 \end{array}$ *6 points*
Which of the following statements are true?

- $BPP \subseteq PP$
- PP is closed under complementation
- $P^{\#P} = P^{PP}$
- If $PP \subseteq \Sigma_i^P$, then PH collapses to Σ_i^P

No, the answer is incorrect.
Score: 0

Accepted Answers:
$BPP \subseteq PP$
PP is closed under complementation
$P^{\#P} = P^{PP}$
If $PP \subseteq \Sigma_i^P$, then PH collapses to Σ_i^P

8) Suppose you knew that $PH \subseteq BPP^{\oplus P}$. Which of the following statements would then imply Toda's theorem? *2 points*

- $NP^{\oplus P} \subseteq P^{\#P}$
- $PP^{\oplus P} \subseteq P^{\#P}$
- $coNP^{\oplus P} \subseteq P^{\#P}$
- $RP^{\oplus P} \subseteq P^{\#P}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$PP^{\oplus P} \subseteq P^{\#P}$