Assignment 3
The due date for submitting this assignment is December 20, 2023.

Due on: Spring 2024

Problem 1:
Let T be a rooted binary tree of width k and height h that represents an arithmetic expression as follows: each leaf node is labeled with an operation in the range 1 to n and each nonleaf node is labeled with an operator. Given T as an input, what is the space bound that can be achieved for evaluating the expression represented by T?

(a) \( O(n) \)
(b) \( O(n \log n) \)
(c) \( O(n \log \log n) \)
(d) \( O(n) \)

Problem 2:
Given an \( n \times n \) matrix that each entry is 0 or 1, what is the time space complexity bound to compute \( A^T \)?

(a) \( O(n) \)
(b) \( O(n^2) \)
(c) \( O(n^3) \)
(d) \( O(n \log n) \)

Problem 3:
Consider the following language:
\( \text{FACTS} = \{ \langle G, x \rangle | G \text{ is a connected graph that contains a cycle of negative weight} \} \)
What is the smallest complexity class that the \text{FACTS} belongs to?

(a) \( \text{P} \)
(b) \( \text{NP} \)
(c) \( \text{PSPACE} \)
(d) \( \text{NPSPACE} \)

Problem 4:
Consider the following language:
\( \text{IPR} = \{ \langle G, x, y \rangle | \text{P \not\leq \text{NP}} \text{ and G is a graph such that at least 50\% \text{ of its edges are involved in a question} \} \}

What is the smallest complexity class that the \text{IPR} belongs to?

(a) \( \text{P} \)
(b) \( \text{NP} \)
(c) \( \text{PSPACE} \)
(d) \( \text{NPSPACE} \)

Problem 5:
Given an undirected graph \( G \) with the best query bound that can be achieved by computing a spanning tree of \( G \):

(a) \( \text{P} \)
(b) \( \text{NP} \)
(c) \( \text{PSPACE} \)
(d) \( \text{NPSPACE} \)

Problem 6:
Suppose we have \( \text{P} \leq \text{NP} \). What can we conclude for \( \text{P}, \text{NP} \) and \( \text{coNP} \)?

(a) \( \text{P} = \text{NP} \)
(b) \( \text{P} \neq \text{NP} \)
(c) \( \text{P} = \text{coNP} \)
(d) \( \text{P} \neq \text{coNP} \)

Problem 7:
Consider the following statements:

(a) \( \text{P} = \text{NP} \)
(b) \( \text{P} \neq \text{NP} \)
(c) \( \text{NP} = \text{coNP} \)
(d) \( \text{NP} = \text{coNP} \)

Problem 8:
Which of the following statement is true for hyperplane reduction \( L \)?

(a) \( L \) is in \( \text{P} \)
(b) \( L \) is in \( \text{NP} \)
(c) \( L \) is in \( \text{coNP} \)
(d) \( L \) is in \( \text{NP} \)