Assignment 4

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2019-09-25, 23:59 IST.

For every ring \( R \), we define a ring \( R^+ \) as the ring of infinite sequences of elements of \( R \). \( R^+ \) consists of all infinite sequences of the form \((a_0, a_1, ..., a_n, ...)\) where \( a_0, a_1, ..., a_n \) are all elements of \( R \). Addition and multiplication in this ring is defined component-wise, i.e.,

\[(a_0, a_1, ..., a_n) + (b_0, b_1, ..., b_n) = (a_0 + b_0, a_1 + b_1, ..., a_n + b_n)\]

and

\[(a_0, a_1, ..., a_n)(b_0, b_1, ..., b_n) = (a_0 b_0, a_1 b_1, ..., a_n b_n)\].

Now, based on this, solve \( Q.1 \) and \( Q.2 \).

1) Which of the following is correct option? 15 points

- For any ring \( R \), every ideal of \( \mathbb{Z}^\infty \) can be expressed as \( \langle f \rangle \), for some \( f \in \mathbb{Z}^\infty \)
- First option is incorrect, but every ideal of \( \mathbb{Z}^\infty \) can be expressed as \( \langle f \rangle \), for some \( f \in \mathbb{Z}^\infty \)
- For any ring \( R \), every ideal of \( \mathbb{Z}^\infty \) can be expressed as \( \langle t_0, t_1, ..., t_k \rangle \), for some \( t_0, t_1, ..., t_k \in \mathbb{Z}^\infty \) where \( k \) is some finite number.
- None of the above.

No, the answer is incorrect.

Score: 0

Accepted Answers:
First option is incorrect, but every ideal of \( \mathbb{Z}^\infty \) can be expressed as \( \langle f \rangle \), for some \( f \in \mathbb{Z}^\infty \)

2) Let \( R \) be some ring. Which of the following is correct?

- \((u_0, u_1, ..., u_m)\) is a unit of \( \mathbb{Z}^\infty \) if \( u_0, u_1, ..., u_m \) are all units of \( R \).
- \((p_0, p_1, ..., p_n)\) is irreducible in \( \mathbb{Z}^\infty \) if \( p_0, p_1, ..., p_n \) are all irreducible in \( R \).
- \( \mathbb{Z}^\infty \) is a unique factorization domain.
- Both (a) and (c).

No, the answer is incorrect.

Score: 0

Accepted Answers:
Both (a) and (c).

3) Which of the following are isomorphic pairs of rings?

1. \( \mathbb{Z}[\sqrt{-5}] / \langle 2, 1 + \sqrt{-5} \rangle \)
2. \( \mathbb{Z}[\sqrt{-5}] / \langle 2 \rangle \)
3. \( \mathbb{Z}[\sqrt{-5}] / \langle 2 \rangle \)
4. \( \mathbb{Z}[\sqrt{-2}] / \langle 2 \rangle \)
5. \( \mathbb{Z}[\sqrt{-2}] \)

- \((1,3)\) and \((2,4)\) are isomorphic pairs
- \((1,3)\) and \((2,5)\) are isomorphic pairs
- \((2,5)\) and \((1,3)\) are isomorphic pairs
- None of them is isomorphic to each other.

No, the answer is incorrect.

Score: 0

Accepted Answers:
Both (a) and (c).

4) Consider the following sets associated with two operations

1. \((E, \cdot)\) where \( E \) is set of even integers
2. \((X, \cdot)\) where \( X \) is set of odd integers
3. \((Q, +)\) where \( Q \) is set of rational numbers

Which of the following statements are correct?

- Only 1 and 2 are commutative ring.
- Only 2 and 3 are commutative ring.
- Only 3 and 1 are commutative ring.
- All three are commutative ring.

No, the answer is incorrect.

Score: 0

Accepted Answers:
Only 3 and 1 are commutative ring.

5) In \( \mathbb{Z}[\sqrt{-3}] \) which of the following statements is true?

- 21 is uniquely factorizable in irreducible products.
- 6 is not uniquely factorizable in irreducible products.
- 21 is not uniquely factorizable in irreducible products.
- Both 6 and 21 is not uniquely factorizable in irreducible products.

No, the answer is incorrect.

Score: 0

Accepted Answers:
Both 6 and 21 is not uniquely factorizable in irreducible products.