1. Show that all the rational functions \( \frac{p(z)}{q(z)} \), for some polynomial \( p(z) \) and \( q(z) \), are analytic over a domain in which \( q(z) \neq 0 \) at every point.

2. Show that \( f(z) = e^{iz^2} \) is an entire function.

3. Study the analyticity of the following functions: \( e^z \) and \( \sin(z) = \frac{e^{iz} - e^{-iz}}{2i} \).

4. Work out the relationship between absolute convergence and uniform convergence.

5. Let \( f \) be a power series with radius of convergence \( R \), then show that for any \( z \) such that \( |z| > R \), \( f \) is absolutely divergent.

6. Show that given the absolutely convergent series
   \[
   A = \sum_{n=0}^{\infty} \alpha_n, \quad B = \sum_{n=0}^{\infty} \beta_n
   \]
   we have the absolutely convergent series
   \[
   AB = \sum_{n=0}^{\infty} \gamma_n, \quad \gamma_n = \sum_{j=0}^{n} \alpha_j \beta_{n-j}.
   \]

7. If \( D \) be a domain bounded by a contour \( C \) for which Cauchy’s theorem is valid and \( f \) is continuous on \( C \) and regular (analytic and single-valued) in \( D \), then show that \( |f| \leq M \) on \( C \) implies \( |f| \leq M \) in \( D \) and if \( |f| = M \) in \( D \), then \( f \) is a constant.
   (Hints: Apply Cauchy’s Integral Formula to \( f(z)^n \) for the first part and to \( \frac{d^n}{dz^n} [f(z)] \) for the second part)