Assignment 9 (Full marks 20)

Q1. The time between two successive breakdowns of a machine can be expressed as
\[ f_X(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}. \]
Over the past few years, the time between two successive breakdowns of the machine are observed to be as follows – 3.5 months, 4.2 months, 6 months, 7.5 months, 7.1 months, 4.5 months, 3 months, 4.3 months, 6.5 months, 7.7 months, 3.6 months, 4.8 months, 6.6 months, 4.7 months, 3.2 months.

What is the value of \( \lambda \)? [2]

(a) 7.51 months  
(b) 5.15 months  
(c) 4.73 months  
(d) 6.53 months

Ans. (b)

Soln.

Using the method of moments, the first moment about the origin of \( f_X(x) \) is

\[ \mu = \frac{1}{\lambda} \int_0^\infty x e^{-\frac{x}{\lambda}} \, dx \]

or, \( \mu = \left[ x e^{-\frac{x}{\lambda}} + \lambda e^{-\frac{x}{\lambda}} \right]_0^\infty \)

or, \( \mu = \lambda \)

Therefore, \( \lambda = \mu = \bar{X} = \frac{1}{15} \sum_{i=1}^{15} x_i = 5.15 \) months.

Ans. 5.15 months.

Q2. A particular meteorological station has 40 years of annual average temperature data. For this sample, the mean and standard deviation are 27.4°C and 2.77°C. What is the 99% confidence interval of the mean temperature? [3]

(a) (24.83; 29.98) °C  
(b) (26.54; 28.26) °C  
(c) (25.44; 29.36) °C  
(d) (26.27; 28.53) °C

Ans. (d)

Soln.
Here $n = 40$. As it is a large sample, $\frac{X - \mu}{S / \sqrt{n}}$ has a standard normal distribution.

To find the 99% confidence interval of the mean temperature, we obtain the value of $z_{0.005}$ for $p = 0.995$ from the standard normal table,

The table gives $z_{0.005} = 2.575$

Now, $\frac{s}{\sqrt{n}}z_{0.005} = \frac{2.77}{\sqrt{40}}(2.575) = 1.13$

The 99% confidence interval of the mean temperature is

$\mu \in (27.4 - 1.13; 27.4 + 1.13) ^\circ C$

i.e, $\mu \in (26.27; 28.53) ^\circ C$

Ans. $(26.27; 28.53) ^\circ C$

Q3. Ten steel rod specimens from a batch are tested. The sample mean of the tensile strength of the rods is found to be 100 MPa. If the sample standard deviation is 15.5 MPa, what is the 99% confidence interval of the mean tensile strength?

(a) $(96.75; 103.25)\text{ MPa}$
(b) $(90.22; 109.77)\text{ MPa}$
(c) $(84.07; 115.93)\text{ MPa}$
(d) $(97.18; 112.82)\text{ MPa}$

Ans. (c)

Soln.

Here $n = 10$.

So, $\frac{X - \mu}{S / \sqrt{n}}$ has a $t$-distribution with $d.o.f = (n - 1) = 9$ degrees of freedom.

To find the 99% confidence interval, we obtain the value of $t_{0.005,9}$ for $\alpha = \frac{1 - 0.99}{2} = 0.005$ and $\nu = 9$ from the $t$-Distribution table,

The table gives $t_{0.005,9} = 3.25$
Now, \( \frac{s}{\sqrt{n}} t_{0.005, 9} = \frac{15.5}{\sqrt{10}} (3.25) = 15.93 \)

The 99% confidence interval of the mean tensile strength is

\[ \langle \mu \rangle_{0.99} = (100 - 15.93; \quad 100 + 15.93) \text{ MPa} \]

\[ i.e, \langle \mu \rangle_{0.99} = (84.07; \quad 115.93) \text{ MPa} \]

Ans. (84.07; 115.93) MPa

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**Q4-1.** From a certain batch of concrete cubes prepared under identical conditions, 35 are selected for testing. It is found that the sample mean compressive strength is 25.4 N/mm\(^2\) and the sample standard deviation is 4.2 N/mm\(^2\). Find the characteristic strength of the concrete. (Characteristic strength of concrete refers to the 95% dependable strength) [2]

(a) 18.49 N/mm\(^2\)
(b) 25.40 N/mm\(^2\)
(c) 17.05 N/mm\(^2\)
(d) 22.48 N/mm\(^2\)

Ans. (a)

Soln.

Here \( n = 35 \).

To find the 95% dependable strength, we obtain the value of the standard normal variate for which the cumulative probability is 0.05

The standard normal table gives \( Z = -1.645 \)

The 95% dependable strength is \( 25.4 - 1.645 	imes 4.2 = 18.49 \) N/mm\(^2\) \( \approx 18.49 \) N/mm\(^2\).

Ans. 23.78 N/mm\(^2\)

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**Q4-2.** In the previous question, what is the 95% confidence interval of the mean compressive strength? [2]

(a) (24.85; 25.79) N/mm\(^2\)
(b) (24.00; 26.79) N/mm\(^2\)
(c) (23.00; 27.00) N/mm\(^2\)
(d) (22.50; 26.90) N/mm\(^2\)

Ans. (b)
Soln. To find the 95% confidence interval, $\alpha=1-0.95=0.05$, $\alpha/2=0.025$

To find the 95% confidence interval, we obtain the value of $z_{0.025}$ for $p = 0.975$ from the standard normal table,

The table gives $z_{0.025} = 1.96$

Now, \[ \frac{s}{\sqrt{n}}z_{0.025} = \frac{4.2}{\sqrt{35}}(1.96) = 1.39 \]

The 95% confidence interval of the mean strength of the concrete cubes is

\[ (25.4 - 1.39; 25.4 + 1.39) \text{ N/mm}^2 \]
\[ i.e., (24.00; 26.79) \text{ N/mm}^2 \]

Ans. The 95% confidence interval of the mean strength of the concrete cubes is

\[ (24.00; 26.79) \text{ N/mm}^2. \]

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Q5. Two independent samples of sizes $n_A=12$ and $n_B=13$ are selected from two different batches (A and B) of concrete cubes. The compressive strength of the samples is tested. It is found that the standard deviations of compressive strength of the samples from batches A and B are 2.6 N/mm$^2$ and 3.9 N/mm$^2$ respectively. It is desired to test the null hypothesis $H_0 : \sigma_A^2 \geq \sigma_B^2$ against the alternative hypothesis $H_a : \sigma_A^2 < \sigma_B^2$ at 0.05 level of significance.

Select the correct statement. [2]

(a) The test statistic is 2.60 and the null hypothesis must be rejected
(b) The test statistic is 2.60 and the null hypothesis cannot be rejected
(c) The test statistic is 2.25 and the null hypothesis must be rejected
(d) The test statistic is 2.25 and the null hypothesis cannot be rejected

Ans. (d)

Soln.

Here $n_A=12$, $n_B=13$

Here null hypothesis is $H_0 : \sigma_A^2 \geq \sigma_B^2$

The alternative hypothesis is $H_a : \sigma_A^2 < \sigma_B^2$

Level of significance $\alpha=0.05$

The criteria for rejection of null hypothesis is $F>2.79$

where $F = \frac{s_B^2}{s_A^2}$
and 2.79 is the value of $F_{0.05}$ for 12 and 11 degrees of freedom respectively.

Now, $F = \frac{s_{\bar{y}}^2}{s_d^2} = \frac{(3.9)^2}{(2.6)^2} = 2.25$

As 2.25 is not greater than 2.79, so the null hypothesis cannot be rejected at the 0.05 level of significance.

**Ans. The test statistic is 2.25 and the null hypothesis cannot be rejected**

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**Q6.** From 25 sample locations in a study area, the mean dry unit weight $(\gamma_d)$ of soil is found to be 17.8 KN/m³ and the standard deviation is found to be 1.95 KN/m³. What is the probability that $\gamma_d$ of the soil lies between 17 and 18 KN/m³? [3]

(a) 0.854
(b) 0.264
(c) 0.675
(d) 0.995

**Ans. (c)**

**Soln.**

Here $n = 25, \mu = 17.8$ KN/m³, $\sigma = 1.95$ KN/m³.

It is required to find the probability that $\bar{X}$ lies between 17 and 18 KN/m³.

Here, $Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$ has a standard normal distribution.

Now, $z_1 = \frac{17 - 17.8}{1.95 / \sqrt{25}} = -2.05$

The corresponding probability value from the standard normal table is 0.020.

Again, $z_2 = \frac{18 - 17.8}{1.95 / \sqrt{25}} = 0.51$

The corresponding probability value from the standard normal table is 0.695

Thus the probability that $\gamma_d$ will lie between 17 and 18 KN/m³ is 0.695−0.020=0.675.

**Ans. 0.675**
Q7. In a factory, 18 out of 100 instruments manufactured are found to be defective. 300 instruments from this factory are shipped to a laboratory. Considering 95% confidence level, find the maximum number of instruments that may be defective in the shipment. [3]

(a) 73
(b) 77
(c) 32
(d) 48

Ans. (a)

Soln.

The proportion of instruments that are defective is

\[ \hat{p} = \frac{18}{100} = 0.18 \]

It is a one-sided test.

We obtain the value of \( z_{0.05} \) from the standard normal table,

The table gives \( z_{0.05} = 1.645 \)

Considering 95% confidence level, the maximum proportion that can be defective in the shipment is

\[ \hat{p} + z_{0.05} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.18 + 1.645 \sqrt{\frac{0.18(1-0.18)}{100}} = 0.243 \]

Thus, the maximum number of equipment that can be defective in the shipment is

\[ 0.243 \times 300 = 72.9 \approx 73 \]

Ans. 73.