Assignment No. 7

[1-1] The annual rainfall depth at a site is normally distributed with a mean of 110cm and coefficient of variation of 25%. What is the probability that annual rainfall is between 80 cm and 150cm?

\[ \text{a. 0.372} \]
\[ \text{b. 0.789} \]
\[ \text{c. 0.672} \]
\[ \text{d. 0.381} \]

Answer: b) 0.789

[1-2] In the previous question, what is 90% dependable annual rainfall in cm? Will 95% dependable rainfall be more than 90% dependable rainfall?

\[ \text{a. 74.8 cm; 95% dependable rainfall be more than 90% dependable rainfall} \]
\[ \text{b. 56 cm; 95% dependable rainfall be more than 90% dependable rainfall} \]
\[ \text{c. 56 cm; 95% dependable rainfall be less than 90% dependable rainfall} \]
\[ \text{d. 74.8 cm; 95% dependable rainfall be less than 90% dependable rainfall} \]

Answer: d) 74.8 cm; 95% dependable rainfall be less than 90% dependable rainfall

[2] Thirty five concrete cubes were tested for compressive strength. The compressive strength of the cubes is provided in following table.

Table 1: Compressive strength of the cubes (in N/mm²)

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<tr>
<td>23.28</td>
<td>26.75</td>
<td>32.43</td>
<td>31.98</td>
<td>29.51</td>
<td>29.53</td>
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<td>30.57</td>
<td>31.29</td>
<td>24.13</td>
<td>27.97</td>
<td>24.34</td>
<td>26.81</td>
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<td>30.96</td>
<td>31.42</td>
<td>29.48</td>
<td>30.71</td>
<td>29.39</td>
<td>26</td>
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<tr>
<td>25.94</td>
<td>29.65</td>
<td>27.81</td>
<td>23.6</td>
<td>26.85</td>
<td>26.4</td>
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<tr>
<td>24.15</td>
<td>32.6</td>
<td>23.67</td>
<td>25.62</td>
<td>30.66</td>
<td>32.19</td>
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What is characteristic strength of concrete i.e. 95% dependable compressive strength? Is this concrete acceptable as M25? (Assume that concrete cube compressive strength follows Gaussian distribution.)

\[ \text{a. 28.17 N/mm²; Not acceptable as M25} \]
\[ \text{b. 28.17 N/mm²; Acceptable as M25} \]
\[ \text{c. 23.43 N/mm²; Not acceptable as M25} \]
\[ \text{d. 23.43 N/mm²; Acceptable as M25} \]

Answer: c) 23.43 N/mm²; Not acceptable as M25

[3] The monthly highest streamflow at a gaging station in river is expected to follow log-normal distribution. The mean monthly highest streamflow is 2000m³/s with standard
deviation of 300 m$^3$/s. Find the probability that monthly highest rainfall is higher than 2800 m$^3$/s.

\[
\begin{align*}
a. & \quad 0.0999 \\
b. & \quad 0.0099 \\
c. & \quad 0.1000 \\
d. & \quad 0.0576 \\
\end{align*}
\]

Answer: b) 0.0099

[4] An engineer constructing a bridge across a river is concerned of the possible occurrence of a flood exceeding 100 m$^3$/s, which can seriously affect his/her work. Historical data show that occurrence of a flood exceeding 100 m$^3$/s is a Poisson process. If a flow of such magnitude is exceeded once in 5 years on average, on the basis of recorded data, what is the chance that the work which is scheduled to last 15 months can proceed without interruption or detrimental effects?

\[
\begin{align*}
a. & \quad 0.221 \\
b. & \quad 0.642 \\
c. & \quad 0.358 \\
d. & \quad 0.779 \\
\end{align*}
\]

Answer: d) 0.779

[5] Hourly air traffic at a small airport follows Gaussian distribution. If probability of hourly traffic being at most 50 is 7% and probability of hourly traffic being higher than 200 is 10%. Find the mean hourly air traffic at the airport?

\[
\begin{align*}
a. & \quad 131 \\
b. & \quad 150 \\
c. & \quad 101 \\
d. & \quad 171 \\
\end{align*}
\]

Answer: a) 131

[6-1] On a particular highway section, the time between successive accidents follows an exponential distribution. If, on an average, an accident occurs in this highway once in 2 years, find the expected time till the fourth occurrence of accident?

\[
\begin{align*}
a. & \quad 4 \\
b. & \quad 7 \\
c. & \quad 8 \\
d. & \quad 10 \\
\end{align*}
\]

Answer: c) 8

[6-2] In the previous question, as per the policy of state if at least 5 accidents occurred within 2 years on a highway section, a road safety review committee will be constituted. What is the probability of formation of committee?
[7-1] An online firm takes an average of 2 days to complete an order. If the time taken to complete an order is an exponential random variable, then what is the probability that the order will not be completed till the 5th day?

a. 0.042
b. 0.327
c. 0.082
d. 0.122

Answer: c) 0.082

[7-2] In the previous question, what is the probability that the order will be completed within 6 days? Moreover, also find the probability that the order will be completed within 6 days given the order is not complete on the 4th day?

a. 0.950 and 0.632
b. 0.950 and 0.950
c. 0.632 and 0.632
d. 0.632 and 0.950

Answer: a) 0.950 and 0.632

[8] In a certain stream, the maximum annual daily discharge has an average of 8000 m$^3$/s and a standard deviation of 900 m$^3$/s. If maximum annual daily discharge follows Gumbel distribution, what is the maximum annual daily discharge with a return period of 25 years?

a. 9526.73 m$^3$/s
b. 10079.85 m$^3$/s
c. 9226.35 m$^3$/s
d. 9835.93 m$^3$/s

Answer: d) 9835.93 m$^3$/s

**Answers**

[1-1] The standard deviation of annual rainfall depth

\[
\sigma = 110 \times \frac{25}{100} = 27.5 \text{ cm}
\]

The probability that annual rainfall is between 80 cm and 150 cm
= \( P(80 \leq X \leq 150) \)

\[
P\left(\frac{80 - 110}{27.5} \leq Z \leq \frac{150 - 110}{27.5}\right)
\]

\[
P(1.45) - P(-1.09) = 0.9265 - 0.1379 = 0.7886
\]

[1-2] Suppose \( r \) is value of 90% dependable annual rainfall.

So, \( P(X \geq r) = 0.9 \)

\[1 - P\left(Z < \frac{r - 110}{27.5}\right) = 0.9\]

\[P\left(Z < \frac{r - 110}{27.5}\right) = 0.1\]

\[r - 110 \quad = \quad -1.28\]

\[r = 110 - 1.28 \times 27.5 = 74.8 \, \text{cm}\]

The 95% dependable annual rainfall will be less than 90% dependable annual rainfall, because the cumulative probability till 95% dependable annual rainfall is 5% (lesser than 10% cumulative probability in case of 90% dependable annual rainfall).

[2] Mean compressive strength of the cube = 28.17 N/mm²

Standard deviation of compressive strength of the cube = 2.88 N/mm²

Characteristics strength of concrete (\( X_c \)) is given by 95% dependable compressive strength i.e. 5% cumulative probability compressive strength, or

\[P\left(Z < \frac{X_c - 28.17}{2.88}\right) = 0.05\]

\[\frac{X_c - 28.17}{2.88} = -1.645\]

\[X_c = 28.17 - 2.88 \times 1.645 = 23.43 \, \text{N/mm}\]²

No, concrete is not acceptable as M25.

[3] Let \( X \) be monthly highest streamflow and \( Y = \ln(X) \). So, \( X \) follows log normal distribution but \( Y \) follows normal distribution.

\[
\bar{X} = 2000 \, m^3 / s \quad \text{and} \quad S_X = 300 \, m^3 / s
\]

Coefficient of variation for \( X = C_v = \frac{S_X}{\bar{X}} = \frac{300}{2000} = 0.15\)

Standard deviation of \( Y = S_Y = \sqrt{\ln(1 + C_v^2)} = \sqrt{\ln(1 + 0.15^2)} = 0.1492\)
Mean of Y = \bar{Y} = \frac{1}{2} \ln \left( \frac{\bar{X}^2}{1 + C_v^2} \right) = \frac{1}{2} \ln \left( \frac{2000^2}{1 + 0.15^2} \right) = 7.590

Probability of monthly highest streamflow exceed 2800 m$^3$/s

\[= P(X > 2800)\]
\[= P(Y > \ln(2800))\]
\[= P(Y > 7.937)\]
\[= 1 - P(Y \leq 7.937)\]
\[= 1 - P(Z \leq \frac{7.937 - 7.590}{0.1492})\]
\[= 1 - P(Z \leq 2.33)\]
\[= 1 - 0.9991\]
\[= 0.0009\]

[4] Let $T$ be the random variable for the time between successive occurrence of a flood.

So, $\bar{T} = 5$ and $\lambda = \frac{1}{\bar{T}} = 0.2$

Then $f_T(t) = 0.2e^{-0.2t}$ and $F_T(t) = P(T \leq t) = 1 - e^{-0.2t}$

The chance that the work which is scheduled to last 15 months can proceed without interruption or detrimental effects

\[= P(T > \frac{15}{12}) = e^{-0.2\times1.25} = 0.7788\]

[5] Let $X$ be random variable for hourly air traffic and $Z$ is corresponding reduced normal variate. Moreover, let us suppose $\bar{X}$ is mean hourly air traffic and $S_X$ is the standard deviation of hourly air traffic.

So, $Z = \frac{X - \bar{X}}{S_X}$

Probability of hourly air traffic being at most 50 = 0.07

\[\Rightarrow P(X < 50) = 0.07\]

\[\Rightarrow P\left(Z < \frac{50 - \bar{X}}{S_X}\right) = 0.07\]

\[\Rightarrow \frac{50 - \bar{X}}{S_X} = -1.48\]

\[\Rightarrow \bar{X} - 1.48S_X = 50\]

- eq(1)

Probability of hourly traffic being higher than 200 = 0.1

Probability of hourly traffic being at most 200 = 0.9
\[ P(X < 200) = 0.9 \]
\[ P \left( \frac{Z}{S_X} < \frac{200 - X}{S_X} \right) = 0.9 \]
\[ \frac{200 - X}{S_X} = 1.28 \]
\[ X + 1.28S_X = 200 \] - eq(2)

By solving eq(1) and eq(2)

\[ \bar{X} = 130.434 \]

As mean hourly air traffic will be an integer so \( \bar{X} = 131 \) planes/hr

[6-1] Time till the fourth accident should be equal to sum of four inter occurrence time for accidents. Time between successive occurrence of accident follow exponential distribution so their sum should follow Gamma distribution with \( \alpha = 4 \) and \( \beta = 2 \) (average time in between successive accidents). So expected time till the fourth accident = \( \alpha \beta = 2 \times 4 = 8 \) years

[6-2] Probability of formation of committee

= Probability that at least 5 accidents occurred in two years

Let \( T \) be time in which five accidents occurs. As distribution of \( T \) will sum of five accident inter-occurrence time (each following exponential distribution), so \( T \) will follow gamma distribution with \( \alpha = 5 \) and \( \beta = 2 \).

\[ f_T(t) = \frac{1}{\beta^\alpha \Gamma(\alpha)} t^{\alpha-1} e^{-\frac{t}{\beta}} = \frac{1}{2^5 \Gamma(5)} t^{5-1} e^{-\frac{t}{2}} = \frac{t^4 e^{-\frac{t}{2}}}{768} \quad t \geq 0 \]

Probability that at least 5 accidents occurred in two years

\[ = P(T \leq 2) = \int_0^2 f_T(t) dt = \int_0^2 \frac{t^4 e^{-\frac{t}{2}}}{768} dt = 0.0037 \]

[7-1] Let \( T \) be the random variable for the time required to fulfil the orders.

So, \( \bar{T} = 2 \) and \( \lambda = \frac{1}{2} = 0.5 \)

Then \( f_T(t) = 0.5e^{-0.5t} \) and \( F_T(t) = P(T \leq t) = 1 - e^{-0.5t} \)

Probability that an order is not completed in 5 days

\[ = P(T > 5) = 1 - P(T \leq 5) = e^{-0.5\times5} = 0.082 \]

[7-2] Probability that an order will be complete within 6 days

\[ = P(T \leq 6) = 1 - e^{-0.5\times6} = 0.950 \]

The probability that order will be complete within 6 days given the order is not complete on 4th day

\[ = P(T \leq 6 | T > 4) \]
Due to memory less property of exponential distribution $P(T \leq 4 + 2 \mid T > 4)$ should be equal to $P(T \leq 2)$, so

$$P(T \leq 6 \mid T > 4) = P(T \leq 2) = 1 - e^{-0.5*2} = 1 - e^{-1} = 0.632$$

[8] Let $X$ is random variable for maximum annual daily discharge.

So, $\bar{X} = 8000 m^3 / s$ \hspace{1cm} $S_X = 900 m^3 / s$

$$\alpha = \frac{S_X}{1.285} = 700.389$$

$$\beta = \bar{X} - 0.5772\alpha = 8000 - (0.5772)(2334.630) = 7595.735$$

The maximum annual daily discharge with a return period of 25 years will have an exceedence probability of $1/25 = 0.04$.

Suppose the value of maximum annual daily discharge with a return period of 25 years is $X_i$, and corresponding reduced variate is $Y_i$.

$$P(X > X_i) = 0.04$$

$$\Rightarrow 1 - e^{-e^{-Y_i}} = 0.04$$

$$\Rightarrow -e^{-Y_i} = \ln(1 - 0.04)$$

$$\Rightarrow e^{-Y_i} = -\ln(0.96)$$

$$\Rightarrow Y_i = -(\ln(-\ln(0.96)))$$

$$Y_i = 3.1985$$

So, $X_i = \alpha Y_i + \beta = 700.389 * 3.1985 + 7595.735 = 9835.93 m^3 / s$