Assignment 2 (Full marks 20)

Q1. A city gets 70% of its required energy from thermal power and remaining from hydropower. If probability of shortage of thermal power is 0.2 and that of hydropower is 0.35, what is the probability of shortage of power to the city? [2]

(a) 0.7  
(b) 0.91  
(c) 0.245  
(d) 0.035  

Ans. (c)

Soln. Let us denote the following events:

S: shortage of power in the city
T: energy from thermal power, H: energy from hydropower

Thus,

\[ P(T) = 0.7 \quad P(S / T) = 0.2 \]
\[ P(H) = 0.3 \quad P(S / H) = 0.35 \]

From total probability theorem, the probability of shortage of power to the city is given by

\[ P(S) = P(S / T)P(T) + P(S / H)P(H) \]
\[ = (0.2)(0.7) + (0.35)(0.3) \]
\[ = 0.245 \]

Q2. There are six concrete blocks out of which three do not fill the criteria of the minimum compressible strength required for construction. If all the blocks are tested, one by one, for their strength, what is the probability that the first three blocks that are tested fail the strength criteria? [2]

(a) \( \frac{1}{2} \)
(b) \( \frac{1}{20} \)
(c) \( \frac{1}{6} \)
(d) \( \frac{1}{8} \)

Ans. (b)

Soln. Let us denote the following events:
A: the first concrete block fails the test
B: the second concrete block fails the test
C: the third concrete block fails the test

\[
P(A \cap B \cap C) = P(A)P(B/A)P(C/A \cap B)
\]
\[
= \frac{3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4} = \frac{1}{20}
\]

Q3. In a certain place, the probability that a cold day will be followed by another cold day is 0.80 and the probability that a warm day will be followed by another warm day is 0.75. Assuming that the weather on a given day depends only on the weather of the previous day, find the probability that the weather on 5 consecutive days are cold, cold, cold, warm and cold respectively. Assume that any given day can be either cold or warm. [2]

(a) 0.36
(b) 0.096
(c) 0.016
(d) 0.048

Ans. (c)

Soln. Let us denote the following events

A: the first day is a cold day
B: the second day is a cold day
C: the third day is a cold day
D: the fourth day is a warm day
E: the fifth day is a cold day

Since any given day can be either cold or warm, and no previous information is available for the first day, \( P(A) = 0.5 \).

For the second and third day, \( P(B) = 0.8, P(C) = 0.8 \).

Since, the probability that a cold day will be followed by another cold day is 0.8, hence the probability that a cold day will be followed by a warm day is \( (1 - 0.8) = 0.2 \).

Thus, \( P(D) = 0.2 \).

Since, the probability that a warm day will be followed by another warm day is 0.75, hence the probability that a warm day will be followed by a cold day is \( (1 - 0.75) = 0.25 \).

Thus, \( P(E) = 0.25 \).
\[
P(A \cap B \cap C \cap D \cap E)
= P(A)P(B/A)P(C/A\cap B)P(D/A\cap B\cap C)P(E/A\cap B\cap C\cap D)
= 0.5 \times 0.8 \times 0.8 \times 0.2 \times 0.25
= 0.016
\]

Q4. Two firms X and Y plan to submit a bid for a construction project. The project contract may be awarded if the bidding amounts are found to be suitable; else the project may be scrapped altogether. Firm X submits the bid. The probability that it will get the contract is 0.75 provided that firm Y does not bid. The probability that Y will bid is 0.75 and if it does, its chance of getting the contract is 0.333.

(i) What is the probability that X will get the contract? [2]
(a) 0.25
(b) 0.547
(c) 0.437
(d) 0.667

Ans. (b)

Soln. Let us denote the following events

A: Firm X gets the contract
B\(_1\): Both firm X and Y bid for the contract
B\(_2\): Firm X bids for the contract but firm Y does not bid for the contract
B\(_3\): The bid gets scrapped

The probability that X will get the contract
\[
P(A) = P(A/B_1)P(B_1) + P(A/B_2)P(B_2)
\]

If \(x\) is the probability that Y gets the contract then,
\[
x = 0.75 \times 0.333
\]
or, \(x = 0.25\)
Thus,
\[
P(A) = P(A/B_1)P(B_1) + P(A/B_2)P(B_2)
= \left[1 - 0.333 - 0.25(1 - x)\right] \times 0.75 + 0.75 \times (1 - 0.75)
= 0.3596 + 0.1875
= 0.5471
\]

(ii) If X gets the contract, what is the probability that Y did not bid? [2]
(a) 0.343
(b) 0.187
(c) 0.25
(d) 0.75

Ans. (a)

Soln. If X gets the contract, then the probability that Y did not bid is

\[
P(B_2 / A) = \frac{P(A / B_1)P(B_2)}{\sum_{i=1}^{2} P(A / B_i)P(B_i)}
\]

\[
= \frac{P(A / B_1)P(B_2)}{P(A / B_1)P(B_1) + P(A / B_2)P(B_2)}
\]

\[
= \frac{0.75 \times (1 - 0.75)}{[1 - 0.333 - 0.25(1 - x)] \times 0.75 + 0.75 \times (1 - 0.75)}
\]

\[
= \frac{0.1875}{0.5471}
\]

= 0.3427

Q5. The following tree diagram shows the conditional probabilities of event A depending on the occurrence and non-occurrence of event B. The non-occurrence of an event is represented by its complement. For example, the non-occurrence of event A is represented as A-complement, which may be denoted as \(A^C\).

(i) In Fig. 2, determine the value of \(P(A)\)
(a) 0.12 
(b) 0.24 
(c) 0.4 
(d) 0.6

**Ans.** (d)

**Soln.** $A$ and $A^c$ are two mutually exclusive events. Also, $B$ and $B^c$ are two mutually exclusive events. Therefore, applying the Bayes theorem,

$$P(A) = P(A \mid B)P(B) + P(A \mid B^c)P(B^c)$$

$$= 0.3 \times 0.4 + 0.8 \times 0.6$$

$$= 0.6$$

**(ii)** In Fig. 2, determine the value of $P(B \mid A)$

(a) 0.2 
(b) 0.3 
(c) 0.6 
(d) 0.8

**Ans.** (a)

**Soln.**

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{\sum_{i=1}^{n} P(A \mid B_i)P(B_i)}$$

$$= \frac{P(A \mid B)P(B)}{P(A \mid B)P(B) + P(A \mid B^c)P(B^c)}$$

$$= \frac{0.3 \times 0.4}{0.3 \times 0.4 + 0.8 \times 0.6}$$

$$= 0.2$$

**(iii)** In Fig. 2, determine the value of $P(B \mid A^c)$

(a) 0.6 
(b) 0.7 
(c) 0.8 
(d) 0.9

**Ans.** (b)

**Soln.**
\[ P(B / A^C) = \frac{P(A^C / B)P(B)}{\sum_{i=1}^{n} P(A^C / B_i)P(B)} \]
\[ = \frac{P(A^C / B)P(B)}{P(A^C / B)P(B) + P(A^C / B^C)P(B^C)} \]
\[ = \frac{0.7 \times 0.4}{0.7 \times 0.4 + 0.2 \times 0.6} \]
\[ = \frac{0.28}{0.28 + 0.12} \]
\[ = 0.7 \]

---

**Q6.** Four measuring instruments are used at a particular station to measure rainfall. 20% of the measurements are done by instrument S\(_1\), wherein a mistake in measurement occurs once in 20 times on an average. 60% of the measurements are done by instrument S\(_2\), wherein a mistake in measurement occurs once in 10 times on an average. 15% of the measurements are done by instrument S\(_3\), wherein a mistake in measurement occurs once in 20 times on an average. 5% of the measurements are done by instrument S\(_4\), wherein a mistake in measurement occurs once in 20 times on an average.

(i) What is the probability that a particular measurement checked at random will be found to be a wrong one? [2]

(a) 0.01
(b) 0.02
(c) 0.0075
(d) 0.08

**Ans.** (d)

**Soln.**

(a) Let \( A_1, A_2, A_3 \) and \( A_4 \) denote the events that the measurements are done by the measuring instruments S\(_1\), S\(_2\), S\(_3\) and S\(_4\) respectively. Let B denote the event that the recorded measurement is wrong.

\[ P(A_1) = \frac{20}{100} = \frac{1}{5} \quad P(B / A_1) = \frac{1}{20} \]
\[ P(A_2) = \frac{60}{100} = \frac{3}{5} \quad P(B / A_2) = \frac{1}{10} \]
\[ P(A_3) = \frac{15}{100} = \frac{3}{20} \quad P(B / A_3) = \frac{1}{20} \]
\[ P(A_4) = \frac{5}{100} = \frac{1}{20} \quad P(B / A_4) = \frac{1}{20} \]
The probability that a particular measurement checked at random will be found to be a wrong one is given by

\[ P(B) = P(B/A_1)P(A_1) + P(B/A_2)P(A_2) + P(B/A_3)P(A_3) + P(B/A_4)P(A_4) \]
\[ = \left(\frac{1}{20}\right)\left(\frac{1}{5}\right) + \left(\frac{1}{10}\right)\left(\frac{3}{5}\right) + \left(\frac{1}{20}\right)\left(\frac{3}{20}\right) + \left(\frac{1}{20}\right)\left(\frac{1}{20}\right) \]
\[ = 0.08 \]

**(ii)** If a particular measurement is found to be wrong, what is the probability that it is recorded by the measuring instrument \( S_1 \)?

(a) \( \frac{1}{8} \)
(b) \( \frac{1}{6} \)
(c) \( \frac{1}{20} \)
(d) \( \frac{3}{8} \)

**Ans.** (a)

**Soln.**

The wrong measurement may have been recorded by either of the measuring instruments \( S_1, S_2, S_3 \) and \( S_4 \). The probability that the wrong measurement is recorded by the measuring instrument \( S_1 \) is given by

\[ P(A_1/B) = \frac{P(B/A_1)P(A_1)}{\sum_{j=1}^{4} P(B/A_j)P(A_j)} \]
\[ = \frac{1}{8} \]