ASSIGNMENT #7

1. The block is allowed to deform in y-direction

$\therefore \epsilon_y = \Delta$; $\epsilon_x = \epsilon_z = 0$; Δ assumed deformation

From Stress Relationship.

$$\sigma_x - \nu(\sigma_y + \sigma_z) = E\epsilon_x = 0 \quad \ldots (1)$$

$$\sigma_y - \nu(\sigma_x + \sigma_z) = E\epsilon_y = E\Delta \quad \ldots (2)$$

$$\sigma_z - \nu(\sigma_x + \sigma_y) = E\epsilon_z = 0 \quad \ldots (3)$$

$$\sigma_x(1+\nu) - \sigma_z(1+\nu) = 0 \qquad \text{Equating (1) and (3).}$$

$$\sigma_x = \sigma_z$$

From (2)

$$\sigma_y - \nu(2\sigma_x) = E\Delta$$

$$\sigma_x = -\frac{E\Delta}{2\nu} + \frac{\sigma_y}{2\nu} \quad \ldots (4)$$

From (4) and (3) and equating $\sigma_x = \sigma_z$.

$$(1-\nu)\left[-\frac{E\Delta}{2\nu} + \frac{\sigma_y}{2\nu}\right] = \nu\sigma_y$$

Solving for σ_y:

$$\sigma_y\left[\nu - \frac{1-\nu}{2\nu}\right] = -\frac{1-\nu}{2\nu}E\Delta$$

$$\therefore \sigma_y = \frac{(1-\nu)}{(1+\nu)(1-2\nu)} \times E\Delta.$$

in terms of F and a, L, c

$$\frac{F_o}{4a^2} = \frac{(1-\nu)}{(1+\nu)(1-2\nu)}\frac{Ec}{L}$$

$$\therefore F_o = \frac{(1-\nu)}{(1+\nu)(1-2\nu)}\frac{4Eca^2}{L}$$

2 $\sigma_x = 130\,MPa$; $\sigma_y = -70\,MPa$; $\tau_{xy} = 80\,MPa$.

$\nu = 0.3$, $E = 205\,GPa$.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{130-70}{2} \pm \sqrt{\left(\frac{130+70}{2}\right)^2 + 80^2}.$$

$$\sigma_{1,3} = 30 \pm 128.06$$

$$\sigma_1 = 158.06\,MPa.$$

$$\sigma_3 = -98.06\,MPa.$$

2. $$\epsilon_x = \frac{1}{E}\left[\sigma_x - u(\sigma_y + \sigma_z)\right]$$

$$= \frac{1}{205\times10^3}\left[103 + 0.3\times70\right] = 736\times10^{-6}.$$

3. $$\epsilon_y = \frac{1}{E}\left[\sigma_y - u(\sigma_x + \sigma_z)\right]$$

$$= \frac{1}{205\times10^3}\left[-70 - 0.3\times130\right] = -531\times10^{-6}$$

4. $$\epsilon_z = \frac{1}{E}\left[\sigma_z - u(\sigma_x + \sigma_y)\right]$$

$$= \frac{1}{205\times10^3}\left[-0.3\times(70+130)\right] = -87\times10^{-6}.$$

5. $$E = 2G(1+u) \quad ; \quad G = \frac{E}{2(1+u)} = \frac{205\times10^3}{2\times(1+0.3)}.$$

$$G = 78.84\,GPa.$$

$$\tau_{xy} = G\gamma_{xy}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$= \frac{80}{205\times10^3} = 1015\times10^{-6}.$$

6. $$\epsilon_{I,III} = \frac{\epsilon_x+\epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}.$$

$$= \frac{[736-531]\times10^{-6}}{2} \pm 10^{-6}\sqrt{\left(\frac{736+531}{2}\right)^2 + \left(\frac{1015}{2}\right)^2}$$

$$= 102.5\times10^{-6} \pm 811.71\times10^{-6}$$

7. $$\epsilon_I = 914.21\times10^{-6}$$

$$\epsilon_{III} = 709.21\times10^{-6}$$

8, 9 P = Internal Pressure.

$$\sigma_\theta = \frac{pr}{t} \; ; \quad \sigma_z = \frac{pr}{2t} = \frac{\sigma_\theta}{2}$$

$$\sigma_r \simeq 0$$

Max Shear Stress Criterio

$$\frac{\sigma_{max} - \sigma_{min}}{2} = \frac{Y}{2}$$

$$\therefore \sigma_{max} - \sigma_{min} = Y$$

$$\sigma_{max} = \sigma_\theta \; ; \quad \sigma_{min} - 0$$

$$\sigma_\theta - 0 = Y$$

$$\sigma_\theta = 250\,MPa \; ; \quad \sigma_z = \sigma_\theta/2$$

$$\sigma_z = \frac{250}{2} = 125\,MPa \; ; \quad \sigma_r = 0$$

$$\epsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \nu(\sigma_r + \sigma_z)\right]$$

$$= \frac{1}{200 \times 10^3}\left[200 - 0.3 \times 125\right] = 1.0625 \times 10^{-3}$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_r + \sigma_\theta)\right] = \frac{1}{200 \times 10^3}\left[125 - 0.3 \times 250\right]$$

$$= 0.25 \times 10^{-3}$$

$\therefore$ % Change in length $= \frac{\delta L}{L} \times 100\% = \epsilon_z \times 100$

$$= 0.25 \times 10^{-3} \times 100 = 0.025\,\%$$

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% Change in Circumference

$= \epsilon_0 \times 100$

$= 1.0625 \times 10^{-3} \times 100$

$= 0.106\%$