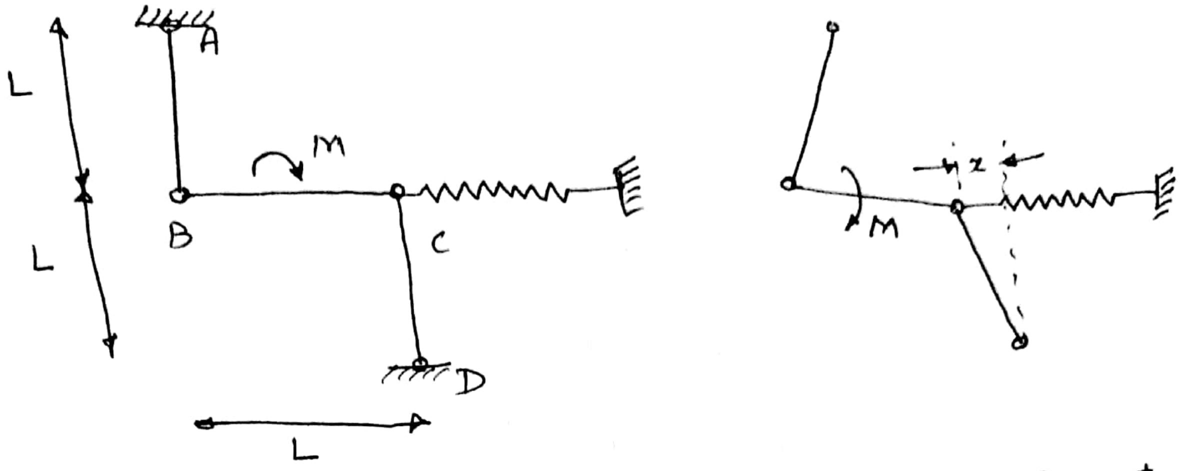
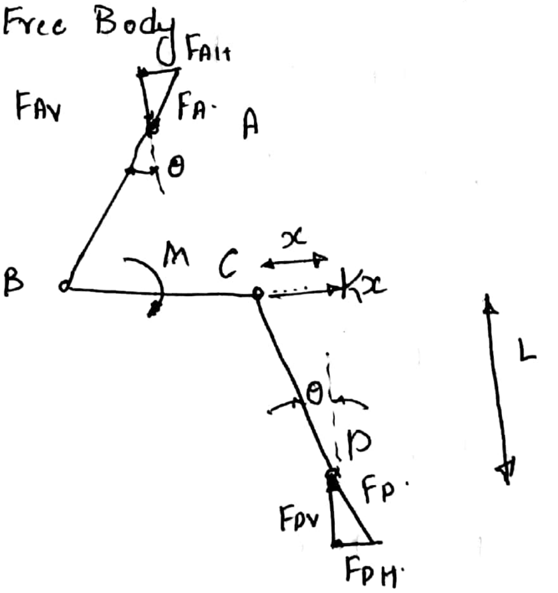
ASSIGNMENT 12

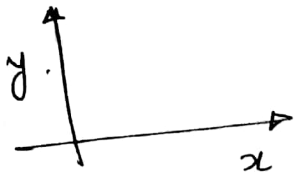
(1)



- Since B, C can transmit no moments, the reactions at A, D must be along AB, DC.

- Consider a Free Body





$$\theta = \frac{x}{L}.$$

- Equilibrium

$\sum M_D = 0$: $(F_{AV} + 2F_{AH})L = M + KxL$ ..... (1).

$\sum F_y = 0$: $F_{AV} = F_{DV}$ .... (2).

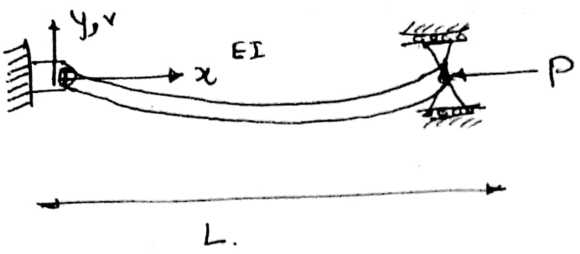
$\sum F_z = 0$: $F_{AH} + F_{DH} = Kx$ ...... (3).

From Geometry $F_{AH} = F_{AV} \dfrac{x}{L}$ ..... (4).

$F_{DH} = F_{DV} \times \dfrac{x}{L}.$

Simplyfying we get $M = \dfrac{KL^2}{2}$

2.



$$EIv'' = M_b = -Pv \quad \ldots\ldots (1).$$

$$EIv'' + Pv = 0$$

$$v'' + \frac{P}{EI} v = 0$$

Let $K^2 = P/EI$.

$$\therefore\ v'' + K^2 v = 0$$

Solution of the differential equation is.

$$v = A\cos Kx + B\sin Kx.$$

Boundary conditions are :- $v(0) = v(L) = 0$

$v(0)$ gives $A = 0$

$v(L) = 0$ we have $B\sin KL = 0$

For non trivial solution

$$\sin KL = 0$$

$$KL = n\pi \qquad , \quad n = 1, 2, 3 \ldots,$$

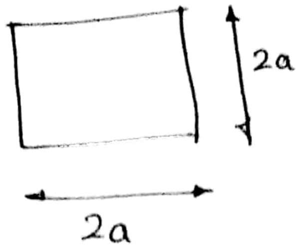
$$\sqrt{\frac{P}{EI}}\, L = n\pi.$$

$$P_{cr} = \frac{n^2\pi^2 EI}{L^2}.$$

For $n = 1$

$$P_{cr} = \frac{\pi^2 EI}{L^2}.$$

3.



For failure by yielding

$P = AY = A\sigma_y$

$A = 2a \times 2a = 4a^2$

$P = 4a^2 \times \sigma_y$

$4a^2 \sigma_y = P$

$a = \sqrt{\frac{P}{4\sigma_y}}$

$a = \sqrt{\frac{50 \times 10^3}{4 \times 315}}$

$a = 6.3 mm.$

$2a = 12.6 mm.$

4. For Buckling

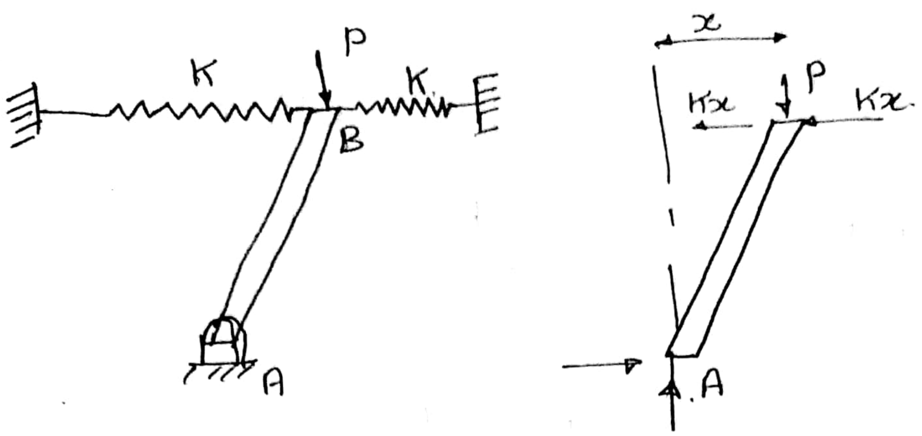
$P_{cr} = \frac{\pi^2}{4} \frac{EI}{L^2}.$

$I = \frac{bh^3}{12} = \frac{2a(2a)^3}{12} = \frac{16a^4}{12} = \frac{4}{3}a^4.$

$a^4 = \sqrt[4]{\frac{3 \times 2^2 \times 50 \times 10^3 \times 10^6}{\pi^2 \times 200 \times 10^3}} = 23.48 mm.$

$2a = 46.96 mm.$

5.



$\sum M_A$ gives.

$$P_{cr} \times x = KxL + KxL$$

$$P_{cr} = 2KL$$

$$P_{cr} = 2 \times 80\,KN/m \times 1$$

$$= 160\,KN.$$

6. When AB behaves as a hinged column,

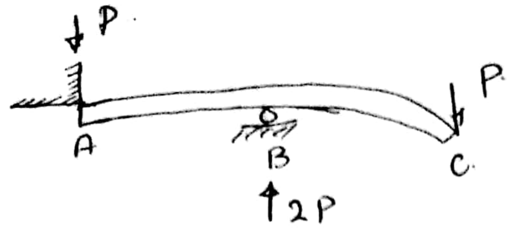
$$P_{criti} = \frac{\pi^2 EI}{L^2}.$$

$$I = \frac{\pi}{64} \times (25 \times 10^{-3})^4$$

$$= \frac{\pi^2 \times 75 \times 10^9 \times \frac{\pi}{64} \times (25 \times 10^{-3})^4.}{1^2}$$

$$= 14.1\,KN.$$

7.



$$EI\frac{d^2y}{dx^2} = -Px + 2P\left(x - \frac{L}{2}\right)$$

$$EI\frac{dy}{dx} = \frac{-Px^2}{2} + P\left(x - \frac{L}{2}\right)^2 + C_1$$

$$EIy = -\frac{Px^3}{6} + \frac{P}{3}\left(x - \frac{L}{2}\right)^3 + C_1x + C_2.$$

Boundary Conditions.

$y(0) = 0$; $\therefore C_2 = 0$

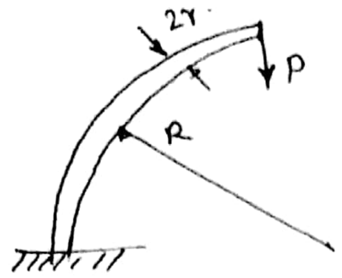
$y\left(\frac{L}{2}\right) = 0$; $C_1 = \frac{PL^2}{24}.$

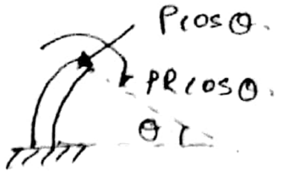
$$EIy = \frac{Px}{6}\left(\frac{L^2}{4} - x^2\right) + \frac{P}{3}\left(x - \frac{L}{2}\right)^3$$

$$y_L = \frac{1}{EI}\left[\frac{PL}{6}\left[\frac{L^2}{4} - L^2\right] + \frac{P}{3}\left[L - \frac{L}{2}\right]^3\right].$$

$$= \frac{-PL^3}{12EI}.$$

8.





$$\bar{U} = \int \frac{F^2}{2AE}\, ds \qquad ; \; A = \pi r^2$$

$$= \int_0^{\pi/2} \frac{P^2 \cos^2\theta}{2AE} \, R \cdot d\theta$$

$$U = \frac{P^2 R}{8 r^2 E} \quad ; \quad \delta a = \frac{\delta U}{\delta P} = \frac{PR}{4 r^2 E}$$

9.

$$\bar{U} = \int \frac{M^2}{2EI}\, ds$$

$$= \int_0^{\pi/2} \frac{P^2 R^2 \cos^2\theta}{2EI} (R\, d\theta)$$

$$U = \frac{P^2 R^3}{2 r^4 E} \quad ; \quad \delta b = \frac{\delta U}{\delta P} = \frac{PR^3}{r^4 E}$$

10.

$$\frac{\delta a}{\delta b} = \frac{r^2}{4R^2}$$