Assignment 7

1) Consider a simple harmonic oscillator given by **1 point**

$\ddot{x} + \omega^2 x = 0$

The trajectories in the phase plane ($x - \dot{x}$) for real ω have the shape of a/an

- circle
- ellipse
- square
- None of the above

Accepted Answers:
ellipse

2) As $t \rightarrow \infty$, the solution to the damped harmonic oscillator given by **1 point**

$$\ddot{x} + 3\dot{x} + x = 0$$

, is approximately proportional to

- $e^{-2.6t}$
- $e^{-0.4t}$
- $e^{-1.5t}$
- e^{-t}

Accepted Answers:
$e^{-0.4t}$

3) For a damped harmonic oscillator with $\gamma = 2, \omega = 2$, the critical point at $x = 0, x = 0$ is a **1 point**

- stable fixed point but not a spiral point

- spiral point but not a stable fixed point
- Neither a stable fixed point nor a spiral point
- Both a stable fixed point and a spiral point

Accepted Answers:
Both a stable fixed point and a spiral point

4) For a simple pendulum parametrized by angle θ, the point $\theta = 3\pi, \dot{\theta} = 0$ corresponds to a/an ***1 point***

- regular point
- stable fixed point
- unstable fixed point
- None of the above

Accepted Answers:
unstable fixed point

5) Linearizing the simple pendulum given by ***1 point***
$\ddot{\theta} + \omega^2 \sin\theta = 0$
near $\theta = 2\pi$ yields trajectories that look like

- ellipses
- hyperbolas
- spirals
- None of the above

Accepted Answers:
ellipses

6) The equation of the separatrix separating periodic and unstable motion of the simple pendulum given by ***1 point***
$\frac{\dot{\theta}^2}{2} - \omega^2 \cos\theta = 4$
is

- $\dot{\theta} = -8\cos(\theta/2)$
- $\dot{\theta} = 8\cos(\theta/2)$
- $\dot{\theta} = 4\cos(\theta/2)$
- None of the above

Accepted Answers:
$\theta = 4\cos(\theta/2)$

7) Linearizing a nonlinear 2nd order ODE about a critical point, and fitting the solutions to the form , it is observed that the real part of both the allowed values of are negative. The critical point is identified as a/an ***1 point***

- stable point
- asymptotically stable point
- unstable point
- None of the above

Accepted Answers:
asymptotically stable point

8) Consider the population dynamics model given by **1 point**

$\dot{x} = 2x - xy$

$\dot{y} = -y + xy$

For this model, one of the critical points is located at the point (x,y) given by

- (0,1)
- (1,1)
- (-1,2)
- None of the above

Accepted Answers:
None of the above

9) The critical point of the ODE $\ddot{x} + 4\dot{x} + 3x = 0$ is **1 point**

- a node
- a stable spiral point
- an unstable spiral point
- None of the above

Accepted Answers:
a stable spiral point

10) Performing linear stability analysis around a critical point, it is observed that both the eigenvalues (λ) are imaginary. The critical point in this case is a/an **1 point**

- node
- center
- spiral
- saddle

Accepted Answers:
center