Unit 6 - Week 5: 2nd Order ODEs, Power Series Method

Assignment 5

1) For which of the DEs below can the general solution be written as a linear combination of two linearly independent solutions ? *1 point*

- $y'' \sin x + y' \cos x + y = 0$
- $y'' + y' \cos x + y^2 = 0$
- $y'' + y' + 3y = 1$
- None of the above

Accepted Answers:
$y'' \sin x + y' \cos x + y = 0$

2) The function $y = e^{3x} \cos x$ satisfies the homogeneous ODE *1 point*

- $y'' + \sin xy' + 3 \cos xy = 0$
- $y'' + 9 \sin xy' + 3 \cos xy = 0$
- $y'' - 6y' + 10y = 0$
- None of the above

Accepted Answers:
$y'' - 6y' + 10y = 0$

3) The function $y = \sin 2x + x$ satisfies the homogeneous ODE *1 point*

- $y'' + 4xy' + 4y = 0$
- $y'' + \frac{4xy'}{2 \cos 2x+1} + 4y = 0$
- $y'' + 4x \sin 2xy' + 4y = 0$
- None of the above

Accepted Answers:
None of the above

4) The function $y_1 = 1 + \cos x$ is one solution of the ODE $y'' \sin x + y' + y \sin x = 0$. The other linearly independent solution is denoted by $y_2 = uy_1$. The function u_2 is given by ***1 point***

- $\int[\ln(1 + \cos x) + \tan(x/2)]dx$
- $exp(\int[2\ln(1 + \cos x)]dx - \tan(x/2))$
- $exp(1 + \cos x - \tan(x/2)$
- None of the above

Accepted Answers:
$exp(\int[2\ln(1 + \cos x)]dx - \tan(x/2))$

5) The general solution of the ODE $y'' + 4y' + 4y + 2e^{-2x} = 0$ is (a and b are arbitrary constants) ***1 point***

- $e^{-2x}(a + (b + 2)x - x^2)$
- $(a - x^2)e^{-2x} + (b + 2)e^{2x}$
- $(a + 2x)e^{2x} + (b - x^2)e^{-2x}$
- None of the above

Accepted Answers:
$e^{-2x}(a + (b + 2)x - x^2)$

6) The ODE $y'' + 5y' + 6y = 0$ corresponds to a/an ***1 point***

- underdamped harmonic oscillator.
- overdamped harmonic oscillator
- critically damped harmonic oscillator
- None of the above

Accepted Answers:
overdamped harmonic oscillator

7) The correct statement regarding the solution of the ODE $(1 - x^2)y'' + 2xy' + y = 0$ is ***1 point***

- x=0 and x=1 are ordinary points of the ODE.
- x=0 and x=1 are regular singular points of the ODE.
- x=0 is an ordinary point, but x=1 is a regular singular point of the ODE.
- None of the above

Accepted Answers:
x=0 is an ordinary point, but x=1 is a regular singular point of the ODE.

8) The correct statement regarding the solution of the ODE $(x^2 - x)y'' + \frac{x}{1-x}y' + \frac{1-x}{x^2}y = 0$ is ***1 point***

- The equation can be solved by the Frobenius method about x=0 but not x=1.
- The equation can be solved by the Frobenius method about x=1 but not x=0.
- The equation can be solved by the Frobenius method about x=0 and x=1.
- The equation cannot be solved by the Frobenius method about x=0 or x=1.

Accepted Answers:
The equation cannot be solved by the Frobenius method about x=0 or x=1.

9) The equation $y'' - 2\alpha xy' + (2E - \alpha)y = 0$, where *E* and α are constants is solved using the power series method with the function $y = \sum_{n=0}^{\infty} a_n x^n$. The recursion relation gives $\frac{a_{n+2}}{a_n}$ equal to **1 point**

- $\frac{n\alpha - E}{(n+1)(n+2)}$
- $\frac{2\alpha + 1 - E}{(n-1)(n)}$
- $\frac{\alpha(2n+1) - 2E}{(n+1)(n+2)}$
- None of the above

Accepted Answers:
$\frac{\alpha(2n+1) - 2E}{(n+1)(n+2)}$

10) The dependent part of the solution of the Schrodinger equation for the quantum mechanical 3D rigid rotor using the power series method is expressed in terms of **1 point**

- Legendre polynomials
- Associated Legendre polynomials
- Hermite polynomials
- None of the above

Accepted Answers:
None of the above