ADVANCED MATHEMATICAL METHODS FOR CHEMISTRY - QUIZ 4 - SOLUTIONS

1. $\left[\frac{d^3y}{dx^3}\right]^2 + \sin x \frac{d^2y}{dx^2} + yx^2 = e^{-x}$

order = 3 degree = 2 (b)

2. $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} = 0$ 3rd order ODE, 3 unknowns in general solution (d)

3. $y'' + 2xy' = \underline{3\sin(xy)}$ Nonlinear equation (c)

4. $y' = -\frac{4xy + x}{2x^2+y}$

$(4xy+x)\,dx + (2x^2+y)\,dy = 0$

$\frac{\partial}{\partial y}(4xy+x) = 4x = \frac{\partial}{\partial x}(2x^2+y)$ Exact differential (b)

5. $(4y-2)\,dx + (3x + x\sin y)\,dy = 0$

$W = \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 4 - 3 - \sin y = 1 - \sin y$

Can find $\frac{W}{M} \rightarrow f(y) \Rightarrow$ can find integrating factor dependent on y only

$\ln \gamma(y) = \int \frac{\sin y - 1}{4y-2}\,dy \rightarrow$ does not match (a), (b), (c).

Answer (d)

6. $\frac{d\vec{x}}{dt} = \begin{bmatrix} 4 & -5 \\ -1 & 1 \end{bmatrix}\vec{x}$ where $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$(4-\lambda)(1-\lambda) - 5 = 0 \Rightarrow \lambda^2 - 5\lambda - 1 = 0$ does not match eigenvalues in (a), (b), (c). So answer (d)

7. $2xy' + 3y^2 = 0$

$\frac{2y'}{y^2} = -\frac{3}{x} \Rightarrow -\frac{2}{y} = -3\ln x + C$

$y(1) = 2 \Rightarrow C = -1$

$y = \frac{2}{3\ln x + 1}$ (b)

8. $\frac{d[A]}{dt} = -2[A] + 3[B]$; $\frac{d[B]}{dt} = [A] - [B]$

$\frac{d\vec{C}}{dt} = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix} \vec{C}$ where $\vec{C} = \begin{bmatrix} [A] \\ [B] \end{bmatrix}$

Eigenvalues: $(-2-\lambda)(-1-\lambda) - 3 = 0$
$\lambda^2 + 3\lambda - 1 = 0$ $\quad \lambda_\pm = -1.5 \pm \sqrt{3.25}$

Eigenvectors: $\begin{bmatrix} 1 \\ \frac{0.5 + \sqrt{3.25}}{3} \end{bmatrix}$, $\begin{bmatrix} 1 \\ \frac{0.5 - \sqrt{3.25}}{3} \end{bmatrix}$

Solution $y = c_1 \begin{bmatrix} 1 \\ \frac{0.5 + \sqrt{3.25}}{3} \end{bmatrix} e^{+\lambda_+ t} + c_2 \begin{bmatrix} 1 \\ \frac{0.5 - \sqrt{3.25}}{3} \end{bmatrix} e^{\lambda_- t}$

$c_1 + c_2 = 1$; $\frac{0.5 + \sqrt{3.25}}{3} c_1 + \frac{0.5 - \sqrt{3.25}}{3} c_2 = 1$

$\Rightarrow c_1 - c_2 = \frac{2.5}{\sqrt{3.25}} \approx 1.39$

$\Rightarrow c_1 = 1.2 \quad c_2 = -0.2$

After 1 unit time : $[A] = 1.2 e^{\lambda_+} - 0.2 e^{\lambda_-}$
$= 1.2 e^{0.3} - 0.2 e^{-3.3}$
$= 1.61$

Answer (b) - closest to 1.2

9. $\frac{d[A]}{dt} = -2[A]^2 t \Rightarrow \frac{1}{[A]} = t^2 + c$

Using initial condition, $c = \frac{1}{2}$

After 5 units ; $\frac{1}{[A]} = 25 + \frac{1}{2} = \frac{51}{2}$

$\Rightarrow [A] = \frac{2}{51} \longrightarrow$ (a)

10. $(x^2 + \cos y)\, dy - (3\tan x + \sin y)\, dx = 0$

Taking derivative of (a) w.r.t. x, we get

$3y^2 y' + \frac{9}{\cos x} \sin x - 3\sin y - 3x\cos y\, y' = 0$

Clearly does not satisfy DE. Similarly (b) and (c) do not satisfy. Answer (d)