Unit 4 - Week 3: Special Functions

Assignment 3

1) A function $y(x)$ satisfies the following:
   $y(x) = -3$ for $-\infty < x < -1$
   $y(x) = 2$ for $-1 < x < 1$
   $y(x) = -1$ for $1 < x < \infty$
   This function is equal to
   
   $3h(x + 1) - 2h(x - 1)$
   $-3h(x + 1) + 2h(x - 1) - 2$
   $-3 + 5h(x + 1) - 3h(x - 1)$
   None of the above

   **Accepted Answers:**
   $-3 + 5h(x + 1) - 3h(x - 1)$

2) A function $y(x)$ satisfies the following:
   $y(x) = 2$ for $-\infty < x < -3$
   $y(x) = 1$ for $-3 < x < 0$
   $y(x) = -1$ for $0 < x < 1$
   $y(x) = 0$ for $1 < x < \infty$
   The derivative of this function is equal to
   
   $\delta(x + 3) + \delta(x) + \delta(1)$
   $3\delta(x + 3) - 2\delta(x) - \delta(1)$
   $2\delta(x + 3) + \delta(x) - \delta(1)$
   None of the above

   **Accepted Answers:**
   None of the above

3) The ratio of $\Gamma(7/2)$ to $\Gamma(1/2)$ is

   $\Gamma(7/2) = \frac{(7/2)(6/2)(5/2)(4/2)(3/2)(2/2)(1/2)}{1/2}$
   $\Gamma(1/2) = \sqrt{\pi}$
   Therefore, the ratio is
   
   $\frac{\Gamma(7/2)}{\Gamma(1/2)} = \frac{(7/2)(6/2)(5/2)(4/2)(3/2)(2/2)(1/2)}{\sqrt{\pi}}$

   **1 point**
4) The value of the integral $\int_{0}^{\infty} x^3 e^{-x^2} dx$ is

- $\sqrt{\pi}/2$
- $\sqrt{\pi}/4$
- $1/16$
- None of the above

**Accepted Answers:**
15/8

5) The correct statement about error function, $\text{erf}(x)$, below is

- $\text{erf}(x) > 0$ for all $x \geq 0$
- $\text{erf}(x)$ is an increasing function of $x$ for $0 < x < \infty$.
- $\text{erf}(x)$ is not a continuous function of $x$ for $0 < x < \infty$
- None of the above

**Accepted Answers:**
None of the above

6) The ranges of values of the spherical polar coordinates $r, \theta, \phi$ are

- $-\infty < r < \infty$, $0 \leq \theta < \pi$, $0 \leq \phi < 2\pi$
- $0 \leq r < \infty$, $0 \leq \theta < \pi$, $0 \leq \phi < 2\pi$
- $-\infty < r < \infty$, $0 \leq \theta < 2\pi$, $0 \leq \phi < 2\pi$
- None of the above

**Accepted Answers:**
$0 \leq r < \infty$, $0 \leq \theta < \pi$, $0 \leq \phi < 2\pi$

7) The ranges of values of the cylindrical polar coordinates $\rho, \theta, z$ are

- $-\infty < \rho < \infty$, $0 \leq \theta < \pi$, $-\infty < z < \infty$
- $0 \leq \rho < \infty$, $0 \leq \theta < 2\pi$, $0 \leq z < \infty$
- $0 \leq \rho < \infty$, $0 \leq \theta < 2\pi$, $0 \leq z < \infty$
Advanced Mathematical Methods for Chemistry - Unit 4 - Week 3: Special Functions

8) The partial derivative of the spherical polar coordinate \( \phi \) with respect to the cartesian coordinate \( x \), denoted by \( \left( \frac{\partial \phi}{\partial x} \right)_{y,z} \), is equal to

- \( \frac{y}{x^2+y^2} \)
- \( -\frac{1}{y} \)
- \( \frac{x}{r^2} \)

None of the above

9) According to kinetic theory of gases, the probability that the absolute value of the \( x \)-component of the velocity, \( |v_x| < \sqrt{2kBT/m} \), is

- \( \text{Erf}(1) \)
- \( \text{Erf}(\sqrt{2}) \)
- \( \text{Erf}(2) \)
- None of the above

10) A certain probability distribution in spherical polar coordinates is given by \( P(r, \theta, \phi) = Nr^2 \cos^2(\theta) e^{-r} \). The value of \( N \) so that this distribution is normalized is

- \( 1/8\pi \)
- \( 1/16\pi \)
- \( 1/32\pi \)
- None of the above

Accepted Answers:
- \( 1/32\pi \)