

Advanced Mathematical Methods for Chemistry

ASSIGNMENT 12 - SOLUTIONS

1) $4-3i = \sqrt{4^2+3^2} e^{i \tan^{-1}(3/4)} = 5 e^{i \tan^{-1}(3/4)} \quad (d)$

2) $e^2+i = \sqrt{e^2+1} e^{i \tan^{-1}(1/e^2)}$. Principal value of $\log \sqrt{e^2+1} e^{i \tan^{-1}(1/e^2)}$
 $= \frac{1}{2} \log(e^2+1) + i \tan^{-1}(1/e^2)$

Answer (d)

3) $\text{Log}(-3) = \log(3 e^{i\pi}) = \log 3 + i\pi \quad (c)$

4) At Poles: $z(z^2+z+1)=0 \Rightarrow z=0, z=-0.5 \pm \sqrt{0.75}i$

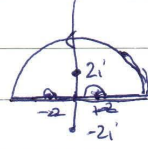
Does not match any of choices

5) Since integrand is analytic within contour, integral = 0 (a)

6) $f(z) = \frac{1+z^2}{z(z-i)} = \frac{(z+i)(z-i)}{z(z-i)} = \frac{z+i}{z}$ $z=i$ is a regular pt. (c)

7) $-2\pi i (-i/2)^2 = +i\pi/2 \quad (c)$
 clockwise

8) $2\pi i \left(\frac{d}{dz} \left(\frac{z+2}{z+0.5} \right) \Big|_{z=0} + \frac{-0.5+2}{(0.5)^2} \right) = 0$

9) $\int_{-\infty}^{\infty} \frac{e^{ix}}{x^2+4} dx = \oint_C \frac{e^{iz}}{z^2+4} dz = \int_{R \rightarrow \infty} \frac{e^{iR e^{i\theta}}}{R^2+4} R d\theta$ 
 $= \frac{2\pi i e^{i \cdot 2i}}{4i} = \frac{\pi}{2e^2} \quad (c)$

10) $\int_{-\infty}^{+\infty} \frac{\sin x}{x^2+3x+3} dx = \text{Im} \int \frac{e^{iz}}{z^2+3z+3} dz$ $\text{Im} = \text{Imaginary part}$
 $= \text{Im} \left[\frac{2\pi i \exp(i(-1.5+0.75i))}{-2 \times 3} \right]$

Answer (d) None of above