1. \[ y'' + \frac{y'}{x} + \frac{y}{x^2} = 0 \]

We can apply the Frobenius method since \( \frac{1}{x} \) does not go to \( \infty \) faster than \( \frac{1}{x} \) and \( \frac{1}{x^2} \) does not go to \( \infty \) faster than \( \frac{1}{x^2} \).

2. \[
y = \sum_{n=0}^{\infty} c_n x^{n+r} \quad y' = \sum_{n=0}^{\infty} c_n x^{n+r-1} \quad y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2} \]

\[
x^2 y'' = \sum_{n=0}^{\infty} c_n (n+r)(n+r-1) x^{n+r} \quad 4y' = \sum_{n=0}^{\infty} (n+r) c_n x^{n+r} \]

\[ r(r-1) + r = 0 \quad r^2 = 0 \]

3. \( x=1 \Rightarrow 1-x^2=0 \) Singular point.

Clearly, it is a regular SP.

So it can be solved by the Frobenius method with \( r \neq 0 \).

4. \[ r(r-1) + r + \frac{9}{4} = 0 \Rightarrow r^2 - \frac{9}{4} = 0 \]

5. \( x=1 \) is not a singular point, so power series method can be used as \( y = \sum_{n=0}^{\infty} a_n (x-1)^n \) i.e. \( r=0 \).

6. Compare with Bessel equation

\[ x^2 y'' + xy' + (x^2 - v^2) y = 0 \]

\[ v^2 = 16 \quad v = 4. \]

One solution involves \( J_4(x) \).

7. \[ R_n(r) \propto r^l L_n, l (r) e^{-r/a_0} \]

\[ m \text{ must be } 0, \text{ not } 1 \text{ since } l=0 \text{. Degree of polynomial } = n-l-1 = 2 \]
\[ \sum_{n=0}^{\infty} a_n x^{n+r} (n+r)(n+r-1) + \sum_{n=0}^{\infty} a_n x^{n+r} (n+r) \]
\[ + \sum_{n=0}^{\infty} a_n x^{n+r+2} - \sum_{n=0}^{\infty} 4a_n x^{n+r} = 0 \]

Coefficient of \( x^n \):
\[ r(r-1) + r = 4 = 0 \]
\[ r^2 + 4 \quad r = \pm 2 \]

If \( r \neq 0 \) Coefficient of \( x^{n+r} \)
\[ (n+r)(n+r-1) a_n + a_{n-2} - 4 : a_n = 0 \]
\[ a_n = -\frac{a_{n-2}}{(n+r)^2 + 4} - \frac{a_{n-2}}{n^2 + 2nr} \]

Consider \( n \) even
\[ a_n = -\frac{a_{n-2}}{n(n+2r)} = (-1)^{n/2} \frac{a_{n-4}}{n(n+2r)(n-2+2r)(n-4+2r)} \]
\[ a_n = \frac{(-1)^{n/2} a_0}{n(n-2)(n-4) \cdots 2 \cdot (n+2r)(n,2r-2) \cdots (2r)} \]

If \( r = 2 \)
\[ a_n = \frac{(-1)^{n/2} a_0}{6^2 \cdot 8^2 \cdot 10^2 \cdots n^2 \cdot (n+2)(n+4)} \cdot 2 \cdot 4 \]
\[ \text{or} \quad a_{2n} = \frac{(-1)^n a_0}{2^{2n} (n!)^2 \cdot (n+2)(n+4)} \]

If \( r = -2 \)
\[ a_0 = \frac{(-1)^n a_0 2}{2^{2n} (n!)^2} \]

9. Solution involves \( J_0 (r/2) \)

10. \( a = \frac{R_0}{n^2 + 2} \) Polynomial \( a \)

\( R_{n, \ell} \sim n^2 \) Polynomial of degree \( n - \ell - 1 \).

Ans. None of above