1. Consider the ODE
\[ \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + y = 0 \]
Solving this ODE using the power series method with \( y = \sum_{n=0}^{\infty} a_n x^n \), we get the recursion relation

(a) \[ a_{n+2} = a_n \frac{2n + 1}{(n+1)(n+2)} \]

(b) \[ a_{n+2} = a_n \frac{2n - 1}{(n+1)(n+2)} \]

(c) \[ a_{n+2} = a_n \frac{n}{(n+1)(n-2)} \]

(d) None of the above
Answer (b)

2. Consider the ODE
\[ \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + y = 0 \]
The condition for a solution to this ODE to be a polynomial of order \( k \) is
(a) \( k = 1 \)  (b) \( k = 2 \)
(c) \( k = \) odd positive integer  (d) No positive integer solution for \( k \)
Answer (d)

3. Consider the ODE
\[ \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + y = 0. \]
This is solved using the power series method by substituting
\[ y = \sum_{n=0}^{\infty} a_n x^n \]
The relation between \( a_4 \) and \( a_0 \) is
(a) \( a_4 = 1/4a_0 \)  (b) \( a_4 = 1/8a_0 \)  (c) \( a_4 = -1/8a_0 \)
(d) None of the above
Answer (c)

4. Using the Rodrigues formula for Legendre Polynomials
\[ P_n(x) = \frac{(-1)^n}{2^n n!} \frac{d^n}{dx^n} (1 - x^2)^n \]
the value of the Legendre Polynomial \( P_5(x) \) is
5. The powers of $x$ that appear in the expression for the Legendre Polynomial $P_6(x)$ are
(a) 0, 2, 4, 6, 8
(b) 0, 2, 4, 8
(c) 2, 4, 6
(d) None of the above
Answer (b)

6. Let $a_0$ and $a_1$ be arbitrary constants. Let $S_{odd}(x)$ and $S_{even}(x)$ denote infinite series in $x$ with only odd and even powers respectively. The general solution of the DE
\[(1 - x^2)\frac{d^2y}{dx^2} + 2x \frac{dy}{dx} + 6y = 0\]
can be expressed in the form
(a) $a_0 P_2(x) + a_1 S_{odd}(x)$
(b) $a_0 S_{even}(x) + a_1 P_1(x)$
(c) $a_0 S_{even}(x) + a_1 P_3(x)$
(d) None of the above
Answer (a)

7. The angular momentum ($|\vec{L}|$) of a rigid rotor with $l = 2, m = 1$ is equal to
(a) 6 $\hbar$ (b) 6 $\hbar^2$ (c) $\sqrt{6}\hbar$ (d) None of the above
Answer (c)

8. The value of
\[\int_{-1}^{+1} P_3(x)P_5(x)dx\]
is equal to
(a) 0 (b) 6 (c) 2 (d) $\sqrt{2}/3$
Answer (a)

9. The value of
\[\int_{-\infty}^{+\infty} H_4(x)H_5(x)dx\]
is equal to
(a) 0 (b) $1920\sqrt{\pi}$ (c) $496\sqrt{\pi}$ (d) None of the above
Answer (d)
10. The polynomial below that solves

\[ \frac{d^2y}{dx^2} - 2x \frac{dx}{dy} + 8y = 0 \]

is

(a) \( x \) (b) \( x^4 - 3x^2 + 2 \) (c) \( 4x^4 - 12x^2 + 3 \)
(d) None of the above

Answer (c)