Mathematics for Chemistry - Assignment 5 - Solutions

1. Nonhomogeneous nonlinear ODE due to $y^2$ term and $xy$ term

2. Linear nonhomogeneous ODE

3. Homogeneous linear ODEs.

4. Nonhomogeneous nonlinear ODEs.

5. 3 arbitrary constants in the general solution of 3rd order ODE and no arbitrary constants in particular solution of 1st order ODE.

6. \[
\frac{dy}{y} = -3x \implies \ln y = -\frac{3x^2}{2} + c
\]
   Put $x=0, y=1 \implies c=0 \implies y = e^{-\frac{3x^2}{2}}$

7. \[
\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1 - (1 - 4y) = 4y
\]
   Clearly \( \frac{1}{N} \left( \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) \) is not a function only of $x$
   Clearly \( \frac{1}{M} \left( \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) \) is not a function only of $y$

   No integrating factor that depends only on $x$ or only on $y$.

8. \[
\frac{\partial M}{\partial y} = 2 = \frac{\partial N}{\partial x} \implies \text{Exact differential}
\]
   \[
u = (2y+3x)dx + (2x+y)dy = 0
\]
   \[
\frac{\partial u}{\partial x} = 2y+3x \implies u = 2xy + \frac{3x^2}{2} + f(y)
\]
   \[
\frac{\partial u}{\partial y} = 2x+dy \implies 2x+y = 2x + \frac{3x^2}{2} + f'{y} \implies f' = \frac{y^2}{2}
\]
   \[
\frac{\partial u}{\partial y} = 2x+dy \implies 2x+y = 2x + \frac{3x^2}{2} + \frac{y^2}{2}
\]
   Solution: $2xy + \frac{3x^2}{2} + \frac{y^2}{2} = C \text{ or } 4xy + 3x^2 + y^2 = C$
9 \quad y' + y = \sin x
\quad y_h = Ae^{-x} \quad y_p = C\sin x + D\cos x

Substituting \( y = y_h + y_p \), we get
\quad C\cos x - D\sin x + C\sin x + D\cos x = \sin x

Comparing \( \cos x \) terms
\quad C + D = 0 \quad C = -D

Comparing \( \sin x \) terms
\quad C - D = 1 \quad D = -\frac{1}{2} \quad C = \frac{1}{2}

\quad y = Ae^{-x} + \frac{1}{2} (\sin x - \cos x)

10 \quad y' + 2y = 3e^{-2x}
\quad y_h = Ae^{-2x} \quad y_p = Bxe^{-2x}

Substituting gives
\quad Be^{-2x} - 2Bxe^{-2x} + 2Bxe^{-2x} = 3e^{-2x}
\quad \Rightarrow B = 3

\quad y = Ae^{-2x} + 3xe^{-2x}