1. Set of all functions $f(x)$ s.t. $\int_{a}^{b} f(x)^2 \, dx = 1$

does not satisfy axioms of vector space since

$$f_2(x) = c \, f(x)$$
does not belong to the space

for any $c \neq 1$. All other sets correspond to real vector spaces.

2. Set (a) is a 4D vector space. Set (c) is not a vector space.
Set (d) is a 3D vector space.
Set (b) is a 2D vector space.

3. For set (a), we have $(c_1, c_2) = \begin{vmatrix} \begin{array}{c} c_1 \\ c_2 \end{array} \end{vmatrix}$

Consider

$$(c_1 + \lambda c_2, c_3) = \begin{vmatrix} \begin{array}{c} (c_1 + \lambda c_2)^* \\ c_3 \end{array} \end{vmatrix}$$

\[= \begin{vmatrix} \begin{array}{c} c_1^* c_3 + \lambda c_2^* c_3 \end{array} \end{vmatrix} \]

But this is not equal to $|c_1^* c_3| + \lambda |c_2^* c_3|$ in general.

\[\therefore (c_1 + \lambda c_2, c_3) \neq (c_1, c_3) + \lambda (c_2, c_3)\]

This is not a real inner product space.

Set (b) is a 4D real inner product space, set (e) is an infinite dimensional inner product space.
Set (d) is a 2D inner product space.

4. Condition (A) is not satisfied for $a = 0$.

5. There can be many choices for basis vectors for a space.

6. Set (b) contains 4 3D vectors and they have to be dependent.
All sets of vectors are linearly independent as can be easily verified. For example, we cannot write
\[ \mathbf{x}^3 = a \mathbf{x} + b \mathbf{x}^2 \] for arbitrary \( a \).
Similarly for the other cases.

8
\[ \mathbf{F} = -\nabla V = -\frac{\partial V}{\partial x} \hat{x} - \frac{\partial V}{\partial y} \hat{y} - \frac{\partial V}{\partial z} \hat{z} \]
\[ -\frac{\partial V}{\partial x} = -x - \frac{1}{2} \frac{x}{(x^2 + y^2 + z^2)^{3/2}} = -x \left( 1 + \frac{1}{r^2} \right) \]
\[ r = \sqrt{x^2 + y^2 + z^2} = \frac{\sqrt{5}}{2} \]
\[ -\frac{\partial V}{\partial y} = -y \times 1.125 \]
\[ -\frac{\partial V}{\partial z} = -z \times 1.125 \]
\[ \mathbf{F} = -1.125 \hat{x} + 1.125 \hat{y} - 1.125 \sqrt{2} \hat{z} \]

9
\[ \mathbf{p} \mathbf{v} = 12 e^{-\frac{x^2 + y^2}{4}} (y \hat{x} + x \hat{y}) \]
\[ \nabla \cdot \mathbf{p} \mathbf{v} = \frac{\partial}{\partial x} \left[ 12 ye^{-\frac{x^2 + y^2}{4}} \right] + \frac{\partial}{\partial y} \left[ 12 xe^{-\frac{x^2 + y^2}{4}} \right] \]
\[ = -12xy e^{-\frac{x^2 + y^2}{4}} \]

10
\[ \nabla \times \mathbf{F} = \left| \begin{array}{ccc}
\hat{x} & \hat{y} & \hat{z} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
yz & xz & xy
\end{array} \right| = \hat{i} \left[ \frac{\partial (xz)}{\partial z} - \frac{\partial (yz)}{\partial y} \right] + \hat{j} \left[ \frac{\partial (yz)}{\partial x} - \frac{\partial (xy)}{\partial z} \right] + \hat{k} \left[ \frac{\partial (xy)}{\partial y} - \frac{\partial (xz)}{\partial x} \right] 
\]
\[ = 0 \]