1) Consider the flow in a channel containing a fluid of density $\rho$ and viscosity $\mu$ of height $h$ and length $L \gg h$ with ends open to atmosphere, as shown in figure 1. The bottom plate is stationary, and the top plate moves with an oscillatory tangential velocity $V = V_0 \cos(\omega t)$. The co-ordinate $x$ is in the stream-wise direction parallel to the plates, and $y$ is perpendicular to the plates. Sufficiently far from the ends of the channel, the flow can be considered unidirectional and fully developed, so that the velocity is independent of the stream-wise $x$ direction. What is the simplest equation for the unidirectional unsteady
Unidirectional transport:
Oscillatory flow in a pipe. Low and high Reynolds number solutions. (unit? unit=76&lesson=77)

Unidirectional transport:
Spherical co-ordinates. Heat conduction from a sphere. (unit? unit=76&lesson=78)

Mass and energy balance equations in Cartesian co-ordinates. (unit? unit=76&lesson=79)

Mass and energy balance equations in Cartesian co-ordinates. Vector notation. (unit? unit=76&lesson=80)

Mass and energy balance equations in spherical co-ordinates. (unit? unit=76&lesson=81)

Mass and energy balance equations in spherical co-ordinates. (unit? unit=76&lesson=82)

Quiz : Week 8 Assessment (assessment? name=117)

Forced convection.

Forced & natural convection.

Fluid density $\rho$, viscosity $\mu$

$\frac{\partial^2 v_x}{\partial y^2} = 0$

$-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2} = 0$

$\rho \frac{\partial v_x}{\partial t} = \mu \frac{\partial^2 v_x}{\partial y^2}$

$\rho \frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$\rho \frac{\partial v_x}{\partial t} = \mu \frac{\partial^2 v_x}{\partial y^2}$

2) What is the Reynolds number which determines whether inertial forces or viscous forces are dominant for the system in Question 1?

$(\rho V_0 h / \mu)$

$(p h^2 / \mu)$

$(\omega h / V_0)$
None of the above.
No, the answer is incorrect.
Score: 0
Accepted Answers:

\((\rho \omega h^2/\mu)\)

3) In Question 1, use the substitution \(v_x = w(y)\text{Real}(\exp(i\omega t))\), where \(\text{Real}()\) is the real part of the complex number, and \(i = \sqrt{-1}\). What is the solution for the velocity field?

\[ v_x = \text{Real}(\exp(y\sqrt{\rho \omega h^2/\mu})\exp(i\omega t)) \]

\[ v_x = \text{Real}\left(\frac{\exp(y\sqrt{\rho \omega h^2/\mu})}{\exp(h\sqrt{\rho \omega h^2/\mu})}\exp(i\omega t)\right) \]

\[ v_x = \text{Real}\left(\frac{\exp(y\sqrt{\rho \omega h^2/\mu}) - \exp(-y\sqrt{\rho \omega h^2/\mu})}{\exp(h\sqrt{\rho \omega h^2/\mu}) - \exp(-h\sqrt{\rho \omega h^2/\mu})}\exp(i\omega t)\right) \]

\[ v_x = \text{Real}\left(\frac{I_0(y\sqrt{\rho \omega h^2/\mu})}{I_0(h\sqrt{\rho \omega h^2/\mu})}\exp(i\omega t)\right) \]

No, the answer is incorrect.
Score: 0
Accepted Answers:

\[ v_x = \text{Real}\left(\frac{\exp(y\sqrt{\rho \omega h^2/\mu}) - \exp(-y\sqrt{\rho \omega h^2/\mu})}{\exp(h\sqrt{\rho \omega h^2/\mu}) - \exp(-h\sqrt{\rho \omega h^2/\mu})}\exp(i\omega t)\right) \]

4) At low Reynolds number, when inertial forces are neglected, what is the velocity profile for the system in Question 1?

\[ \left(\frac{V^2}{h}\right)\sin(\omega t) \]

\[ \left(\frac{V^2}{h}\right)\cos(\omega t) \]

\[ V_0\left(\frac{y}{h} - \frac{y^2}{2h^2}\right)\sin(\omega t) \]

\[ V_0\left(\frac{y}{h} - \frac{y^2}{2h^2}\right)\cos(\omega t) \]

No, the answer is incorrect.
Score: 0
Accepted Answers:
5) In the limit of high Reynolds number, the flow in Question 1 is expected to be restricted to a thin region near the top surface. What is the thickness of the region?

- \( h \)
- \( \sqrt{\rho \omega / \mu} \)
- \( \sqrt{\mu / (\rho \omega)} \)
- \( \mu / (\rho \omega) \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
\( \sqrt{\mu / (\rho \omega)} \)

6) In a spherical co-ordinate system with radial, azimuthal and meridional co-ordinates \((r, \theta, \phi)\) respectively, the expression for the gradient of a vector \( \mathbf{A} \) is,

\[
\nabla \mathbf{A} = \frac{1}{r} \frac{\partial \mathbf{A}_r}{\partial r} + \frac{1}{r \sin(\theta)} \frac{\partial \mathbf{A}_\theta}{\partial \theta} + \frac{1}{r \sin(\theta)} \frac{\partial \mathbf{A}_\phi}{\partial \phi}
\]

None of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:
None of the above

7) The expression for the Laplacian of a scalar \( T \), for the same co-ordinate system as in Question 1, is,

\[
\nabla^2 T = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left( \sin(\theta) \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2 T}{\partial \phi^2}
\]
8) Consider a cylindrical co-ordinate system shown in figure 2 in which the co-ordinates are $r$, the distance from the $z$ axis, the angle $\theta$ made by the projection of the radius vector in the $x - y$ plane with the $x$ axis, and $z$, the distance along the $z$ axis. What is the differential volume in the cylindrical co-ordinate system?

$$dV = r \, dr \, d\theta \, dz$$

9) What is the divergence of a vector $\mathbf{A}$ with components $(A_r, A_\theta, A_z)$ in the cylindrical co-ordinate system as described in Question 8?
\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} &= 0 \\
\frac{\partial \rho u}{\partial x} + \frac{\partial \tau_{ux}}{\partial y} + \frac{\partial \tau_{uy}}{\partial z} &= -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \mu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) + f_x \\
\frac{\partial \rho v}{\partial y} + \frac{\partial \tau_{vx}}{\partial x} + \frac{\partial \tau_{vy}}{\partial z} &= -\frac{\partial p}{\partial y} + \frac{\partial}{\partial y} \left( \mu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) + f_y \\
\frac{\partial \rho w}{\partial z} + \frac{\partial \tau_{wx}}{\partial x} + \frac{\partial \tau_{wy}}{\partial y} &= -\frac{\partial p}{\partial z} + \frac{\partial}{\partial z} \left( \mu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) + f_z
\end{align*}
\]