Unit 14 - Week 9: Estimation Theory

Week 9 Assignment

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2018-03-31, 23:59 IST

1) If \( v[k] = c + e[k] \), where \( e[k] \sim \mathcal{N}(0, \sigma^2) \) then the statistically unbiased least square estimate of the variance of \( v[k] \) with \( N \) is

\[
\frac{1}{N^2} \sum_{k=1}^{N} (v[k] - \frac{1}{N} \sum_{k=1}^{N} v[k])^2
\]

\[
\frac{1}{N} \sum_{k=1}^{N} (v[k] - \frac{1}{N} \sum_{k=1}^{N} v[k])^2
\]

\[
\frac{1}{N-1} \sum_{k=1}^{N} (v[k] - \frac{1}{N} \sum_{k=1}^{N} v[k])^2
\]

\[
\frac{1}{N+1} \sum_{k=1}^{N} (v[k] - \frac{1}{N} \sum_{k=1}^{N} v[k])^2
\]

a. 

b. 

c. 

d.

No, the answer is incorrect.
Score: 0
Accepted Answers: 

2) The standard deviation for two different Gaussian, independent random processes and \( e_1[k] \) known to be \( \sigma_1 \) and \( \sigma_2 \) respectively. In order to obtain equal amount of information about mean for both the processes the ratio of the data points \( \frac{N_1}{N_2} \) should be

\[
\frac{\sigma_1}{\sigma_2}
\]

\[
\frac{\sigma_1^2}{\sigma_2^2}
\]

\[
\frac{\sigma_1 \sigma_2}{\sigma_2^2}
\]

\[
\frac{\sigma_1 \sigma_2}{\sigma_1^2}
\]

a. 

b. 

c. 

d.

No, the answer is incorrect.
Score: 0
Accepted Answers: 

3)
3. If \( v[k] \) is a white noise process with exponential distribution \( f(v[k] = x) = \lambda e^{-\lambda x} \) then \( N \) observations of \( v[k] \) choose the right option(s).

a. There exists \( \lambda \) s.t. \( \sigma^2_{\lambda} = I^{-1}(\lambda) \), \( \lambda = \frac{1}{N} \sum_{k=1}^{N} v[k] \)

b. There exists \( \frac{1}{\lambda} \) s.t. \( \sigma^2_{\frac{1}{\lambda}} = I^{-1}(\frac{1}{\lambda}) \), \( \frac{1}{\lambda} = \frac{1}{N} \sum_{k=1}^{N} v[k] \)

c. There exists \( \frac{1}{\lambda^2} \) s.t. \( \sigma^2_{\frac{1}{\lambda^2}} = I^{-1}(\frac{1}{\lambda^2}) \), \( \frac{1}{\lambda^2} = \frac{1}{N} \sum_{k=1}^{N} v[k] \)

d. None of these.

where \( I(\theta) \) and \( \hat{\theta} \) are the Fisher's Information and estimator of \( \theta \) respectively.

No, the answer is incorrect.
Score: 0
Accepted Answers: b

4. Given the data generating process

\[ v[k] = 0.6 c[k-1] + c[k] \]

use Monte-Carlo simulations and report the variance of the sample variance estimator. Use \( R = 10^5 \) realizations with \( N = 100 \) observations per realization and round off answer to 2 decimal places.

a. 0.5
b. 0.05
c. 1

None of these.

No, the answer is incorrect.
Score: 0
Accepted Answers: b

5. You are the visiting quality inspector of a sport equipment manufacturer. The manufacturer has developed a new, synthetic fishing line that they claim has a mean breaking strength of 8 kg, with a standard deviation of 0.5. You decide to test this claim by randomly selecting 50 fishing lines. You find that the mean breaking strength calculated from this sample is 7.85 kg. At a level of 0.05, would you reject the claim that the manufacturer has made?

a. Yes
b. No

No, the answer is incorrect.
Score: 0
Accepted Answers: a
6. The lower bound for the variance of an unbiased estimator of \( f(\theta) \): a function of a parameter \( \theta \) is
   \[
   \frac{(f'(\theta))^2}{f(\theta)^2}
   \]
   \[
   \frac{f'(\theta)}{f(\theta)}
   \]
   \[
   \frac{f'(\theta)}{f(\theta)}
   \]
   \[
   \frac{(f'(\theta))^2}{f(\theta)^2}
   \]
   No, the answer is incorrect.
   Score: 0
   Accepted Answers:
   a

7. If the point estimates of mean for a random process \( v[k] \) using two different estimators from an \( N \) point dataset are obtained as \( \bar{v}_1 = \frac{1}{N} \sum_{k=1}^{N} v[k] \) and \( \bar{v}_2 = \frac{1}{N-p} \sum_{k=1}^{N} v[i] \) respectively, where \( p > 0 \) choose the correct option(s).
   a. \( \bar{v}_1 \) is statistically and asymptotically unbiased, \( \bar{v}_2 \) is asymptotically unbiased.
   b. \( \bar{v}_1 \) is asymptotically unbiased, \( \bar{v}_2 \) is statistically biased.
   c. \( \bar{v}_1 \) is statistically unbiased, \( \bar{v}_2 \) is statistically unbiased.
   d. \( \bar{v}_1 \) is statistically biased, \( \bar{v}_2 \) is asymptotically unbiased.
   No, the answer is incorrect.
   Score: 0
   Accepted Answers:
   a

8. Given three observations \( v[1] \), \( v[2] \) and \( v[3] \) of a series, the maximum order \( p \) of the process of the form \( v[k] = \sum_{i=1}^{p} d_i v[k-i] + e[k] \) that can be fit and the least square estimates of the corresponding parameters are obtained as __________.
   a. \( p = 1, d_1 = \frac{v[1]}{v[2]} \)
   b. \( p = 2, d_1 = \frac{v[1]}{v[2]}, d_2 = \frac{v[3]}{v[2]} \)
   c. \( p = 2, d_1 = -\frac{v[1]^2}{v[2]}, d_2 = \frac{v[3]}{v[2]} \)
   No, the answer is incorrect.
   Score: 0
9. Consider the data in the table given below:

<table>
<thead>
<tr>
<th>y</th>
<th>198.97</th>
<th>243.02</th>
<th>296.83</th>
<th>362.54</th>
<th>442.81</th>
<th>540.85</th>
<th>660.60</th>
<th>806.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>2.3</td>
<td>2.4</td>
<td>2.5</td>
<td>2.6</td>
<td>2.7</td>
<td>2.8</td>
<td>2.9</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Suppose you are given the predictor expression for \( y \) as

\[
\hat{y} = ae^{bx}
\]

obtain an estimate of \( a \) using linear least squares (round off your answer to 2 decimal places)______.

a. 2.26  
b. 1.5  
c. 1.65  
d. 1.83  

No, the answer is incorrect.  
Score: 0

Accepted Answers:

10. The data given in w9.q10.RData is generated by mixing deterministic and random with the model structure given below:

\[
y[k] = ax[k] + w[k]
\]

\[
w[k] = be[k - 1] + e[k]
\]

where \( e[k] \sim \mathcal{N}(0, \sigma_e^2) \). Using R, find the values of \( \hat{a} \) and \( \hat{b} \).

a. \( \hat{a} = 0.5570, \hat{b} = 0.5864 \)  
b. \( \hat{a} = 0.5714, \hat{b} = 0.5544 \)  
c. \( \hat{a} = 0.5874, \hat{b} = 0.5138 \)  
d. \( \hat{a} = 0.5930, \hat{b} = 0.5064 \)

No, the answer is incorrect.  
Score: 0

Accepted Answers:

a