# Problem Sheet 03

**Problem 1: Number of Flops for Gauss Elimination**

Consider the following system of equations

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 6 \\ 3x_1 - x_2 + x_3 - x_4 &= 3.5 \\ 10x_1 + 2x_2 - x_3 + 3x_4 &= 9.5 \\ -3x_1 - 6x_2 + 4x_3 + 7x_4 &= 3.5 \end{aligned}$$

Solve the equations using Gauss Elimination followed by Back Substitution. Clearly show all the steps and indicate the number of addition/subtraction and multiplication/division operations. Hence compute the total number of Flops (<u>Fl</u>oating Point <u>Op</u>erations) for elimination and back substitution steps. Verify that the number of flops is as shown in the video lectures and/or textbook (Chapra and Canale, Numerical Methods for Engineers).

**Problem 2: Ill-Conditioned System**

Consider the following equations (Problem 8.7 of the Textbook)

$$\begin{aligned} 0.5x_1 - x_2 &= -9.5 \\ 1.02x_1 - 2x_2 &= -18.8 \end{aligned}.$$

- Solve the equations graphically
- Compute the determinant of the "A" matrix. Based on the graphical solution and the determinant, how would you infer that the system is ill conditioned?
- Solve the equation by Gauss Elimination with Back-substitution
- If $a_{11}$ is modified to 0.52, what is the solution of the equations? Interpret these results.

**Problem 3: Round-off Errors and Pivoting**

Consider the linear equations:

$$\begin{aligned} 0.00030x_1 + 3.00000x_2 &= 2.00010 \\ 1.00000x_1 + 1.00000x_2 &= 1.00000 \end{aligned}$$

Solve the above equation using: Gauss Elimination (i) naïve (without pivoting) and (ii) Gauss Elimination with pivoting.

In both the cases, retain five digits after the decimal point. Use chopping and not round-off to remove the additional digits.

**Problem 4: Various Direct Methods with and without Pivoting**

Consider the following linear equations:

$$\begin{aligned} 5x_1 + 2x_2 + 2x_3 &= 2 \\ -3x_1 + 5x_2 - x_3 &= 1 \\ x_1 + 2x_2 - x_3 &= 2 \end{aligned}$$

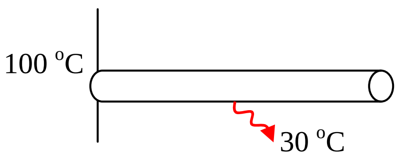
1. Use Gauss Elimination to obtain a lower triangular matrix, L.
2. As discussed in the video lectures, use the coefficients of Gauss Elimination to construct the upper triangular matrix, U.
3. Verify that these matrices satisfy the equation A = LU.
4. Find the determinant of A. Is there another way of obtaining the determinant from either L or U matrices?
5. Use LU decomposition with forward and backward substitutions to solve the above equations.

**Problem 5: Iterative methods**

Solve the equations in Problem 4 using (i) Gauss Siedel method; and (ii) Jacobi iterations.

**Problem 6: Heat conduction in a Metal Rod**

Consider a metal rod which has its one end connected to a hot equipment and the other end exposed to atmosphere such that it dissipates heat to the surroundings. Such structures are routinely used to manage heat: in car radiators, refrigerators, computer chips etc. We will model just one single rod of the assembly and solve it using an appropriate solver.




With appropriate derivation, the model for this system can be represented as:

$$\frac{d^2T}{dz^2} = 0.1(T - 30) \quad \text{subject to} \quad T_{(z=0)} = 100; T_{(z=5)} = 30$$

We discretize the above equation into 10 intervals. Thus, we will get *linear equations* in 51 variables. The actual derivation will be considered towards the end of the computational techniques course (we defer further discussion until that point).

The linear set of equations that you need to solve is as follows:

$$T_1 = 100$$

$$100T_{i-1} - 200.1T_i + 100T_{i+1} = -3 \qquad \text{for } i = 2 \text{ to } 10$$

$$T_{11} = 30$$

With this, we define a vector containing all the temperatures as: $x = [T_1, T_2, \cdots, T_{11}]^T$. The above equations can then be written in the form:

$$Ax = b$$

Here, $A$ is a tri-diagonal matrix.

1. Formulate the problem appropriately in the form $Ax = b$.
2. Solve the above problem using Tri-Diagonal Matrix Algorithm.

**Problem 7: Heat conduction using Gauss-Siedel**

Solve Problem 6 using either Gauss Siedel method or Jacobi iterations.