Assignment 6

Due: 2023-03-17, 20:00 HKT

Instructions: Follow the instructions given in the assignment template.

1. A fraction is said to be in lowest terms if its numerator and denominator are relatively prime. For example, the fraction 3/4 is in lowest terms, but the fraction 6/8 is not.

2. Consider the function f(x) = 3x^2 - 2x + 1. Find the derivative of f(x) and evaluate it at x = 2.

3. Consider the matrices A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} and B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}. Compute AB, BA, and A^2.

4. Consider the following optimization problem:

\[ \text{Minimize } f(x, y) = x^2 + y^2 \]

Subject to

\[ g(x, y) = x + y - 1 \leq 0 \]

5. Suppose we have a function f(x) that is defined on the interval [0, 1]. Suppose we have a point \( a \) in this interval, and we want to approximate f(x) near a. We can use a Taylor polynomial to do this.

6. Consider the following differential equation:

\[ \frac{dy}{dx} = y^2 - 1 \]

with the initial condition y(0) = 1. Find the solution to this differential equation.

7. Consider the function f(x) = x^3 - 3x + 1. Find the critical points of f(x) and determine whether they correspond to local maxima, local minima, or neither.

8. Consider the following system of linear equations:

\[ \begin{align*}
2x + 3y &= 7 \\
4x - y &= 3
\end{align*} \]

Solve for x and y.

9. Consider the following problem:

\[ \text{Maximize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the maximum value of f(x, y) subject to these constraints.

10. Consider the following problem:

\[ \text{Minimize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the minimum value of f(x, y) subject to these constraints.

11. Consider the following problem:

\[ \text{Maximize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the maximum value of f(x, y) subject to these constraints.

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\[ \text{Minimize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
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Find the minimum value of f(x, y) subject to these constraints.

13. Consider the following problem:

\[ \text{Maximize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the maximum value of f(x, y) subject to these constraints.

14. Consider the following problem:

\[ \text{Minimize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the minimum value of f(x, y) subject to these constraints.

15. Consider the following problem:

\[ \text{Maximize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the maximum value of f(x, y) subject to these constraints.

16. Consider the following problem:

\[ \text{Minimize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the minimum value of f(x, y) subject to these constraints.

17. Consider the following problem:

\[ \text{Maximize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the maximum value of f(x, y) subject to these constraints.

18. Consider the following problem:

\[ \text{Minimize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the minimum value of f(x, y) subject to these constraints.

19. Consider the following problem:

\[ \text{Maximize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the maximum value of f(x, y) subject to these constraints.

20. Consider the following problem:

\[ \text{Minimize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the minimum value of f(x, y) subject to these constraints.

21. Consider the following problem:

\[ \text{Maximize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the maximum value of f(x, y) subject to these constraints.

22. Consider the following problem:

\[ \text{Minimize } f(x, y) = x^2 + y^2 \\
\text{Subject to } g(x, y) = x + y - 3 \leq 0 \\
\text{and } h(x, y) = x^2 + y^2 \leq 4 \]

Find the minimum value of f(x, y) subject to these constraints.