Assignment 12

1. Solve the following linear programming problem.

Minimize: \[ f(x) = 2x_1 + 3x_2 \]
Subject to:
\[ x_1 + 2x_2 \leq 10 \]
\[ 2x_1 + x_2 \leq 15 \]
\[ x_1, x_2 \geq 0 \]

The optimal solution is \( x_1 = \frac{10}{3} \) and \( x_2 = \frac{5}{3} \). The optimal value is \( f(x) = \frac{40}{3} \). The optimal solution is \( x = (\frac{10}{3}, \frac{5}{3}) \). The optimal value is \( f(x) = \frac{40}{3} \).

2. Solve the following linear programming problem.

Minimize: \[ f(x) = x_1 + 2x_2 \]
Subject to:
\[ x_1 + x_2 \leq 6 \]
\[ 2x_1 + 3x_2 \leq 9 \]
\[ x_1, x_2 \geq 0 \]

The optimal solution is \( x_1 = 3 \) and \( x_2 = 0 \). The optimal value is \( f(x) = 3 \). The optimal solution is \( x = (3, 0) \). The optimal value is \( f(x) = 3 \).

3. Consider a job shop with three jobs being processed at a constant rate. The total time for the three jobs is 30 hours. Let the total number of jobs be 10. The estimate of the total hours is 30 hours. The optimal solution is \( x_1 = 5 \), \( x_2 = 5 \). The optimal value is \( f(x) = 0 \).

4. Solve the following Non-Linear Programming Problem.

Maximize: \[ f(x) = x_1^2 + 2x_2 \]
Subject to:
\[ x_1 + x_2 \leq 4 \]
\[ x_1, x_2 \geq 0 \]

The optimal solution is \( x_1 = 1 \), \( x_2 = 3 \). The optimal value is \( f(x) = 7 \). The optimal solution is \( x = (1, 3) \). The optimal value is \( f(x) = 7 \).

5. Solve the following Non-Linear Programming Problem.

Minimize: \[ f(x) = x_1^2 + x_2^2 \]
Subject to:
\[ x_1 + x_2 \leq 4 \]
\[ x_1, x_2 \geq 0 \]

The optimal solution is \( x_1 = 1 \), \( x_2 = 1 \). The optimal value is \( f(x) = 2 \). The optimal solution is \( x = (1, 1) \). The optimal value is \( f(x) = 2 \).

6. Solve the following Linear Programming Problem.

Minimize: \[ f(x) = 2x_1 + 3x_2 \]
Subject to:
\[ x_1 + x_2 \leq 5 \]
\[ x_1, x_2 \geq 0 \]

The optimal solution is \( x_1 = 2 \), \( x_2 = 3 \). The optimal value is \( f(x) = 11 \). The optimal solution is \( x = (2, 3) \). The optimal value is \( f(x) = 11 \).

7. Solve the following Linear Programming Problem.

Minimize: \[ f(x) = x_1 + 2x_2 \]
Subject to:
\[ x_1 + x_2 \leq 6 \]
\[ 2x_1 + 3x_2 \leq 9 \]
\[ x_1, x_2 \geq 0 \]

The optimal solution is \( x_1 = 3 \), \( x_2 = 0 \). The optimal value is \( f(x) = 3 \). The optimal solution is \( x = (3, 0) \). The optimal value is \( f(x) = 3 \).

8. Solve the above LP with initial point \( x = (1, 1) \). The optimal value is \( f(x) = 5 \).

9. A manufacturer makes two types of products: A and B. Three machines are used in the process and the time (in minutes) required for each product on the machine is given below:

<table>
<thead>
<tr>
<th>Machine</th>
<th>Product A</th>
<th>Product B</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>M2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>M3</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

Each product is available in a maximum of 100 hours per day. If the profit on each product is \( \$10 \) for type A and \( \$15 \) for type B, and the maximum profit that can be achieved is \( \$1500 \), find the maximum profit that can be achieved in a day.

The solution might be \( x = (50, 50) \).