Week 7 Assignment 7

Due on 2023-11-10 23:59:59

1. Linear interpolation and the second-order central-differencing scheme are applied to discretize the advection and diffusion fluxes, respectively, of the 2D Euler equations. The relevant parameters are $\Delta x = \Delta y = 0.1$, $\Delta t = 0.01$, and $c = 0.1$. The coefficients $c_x$ and $c_y$ for linear interpolation and $\alpha$ for second-order central-differencing are:

- $c_x = 0.5c$ and $c_y = 0.5c$
- $\alpha = 0.5c$

2. Distance between two neighboring nodes $\tilde{a} = \tilde{b} = 0.5$. The interface $a$ is a distance of $0.05$ from $\tilde{a}$ and $0.05$ from $\tilde{b}$, while the value of the source term at point $a$ is $0.2$. The value of the source term at point $c_1$ is $0.1$, and $c_2 = 0.0$. For the discrete form of the acoustic equation at the first node $c_1$, the values close to the boundary $\tilde{a}$ are:

- $a_1 = 0.16667$, $a_2 = 0.16667$, $a_3 = 0.16667$, $a_4 = 0.16667$, $a_5 = 0.16667$

3. In the 2D Euler equations, the advection terms are calculated using forward-difference (FD) schemes, and the diffusive fluxes are calculated using central-difference (CD) schemes, respectively. If $\Delta x = \Delta y = 0.1$, $\Delta t = 0.01$, and $c = 0.1$, then values of $\delta^f$ should be adjusted for $c_x$ and $\alpha$ for any first-order spatial accuracy (e.g., non-dissipative central-differencing):

- $c_x = 0.5c$, $c_y = 0.5c$
- $\alpha = 0.5c$

4. A hybrid differencing where a scheme is applied to discretize the advective flux and a CD scheme is applied to discretize the diffusive flux is called a first-order convective-diffusion scheme (CDF). The relevant parameters are $\alpha = 0.5$, $\tilde{a} = 0.5$, and $\tilde{b} = 0.5$. The coefficients $c_x$ and $c_y$ for any first-order convective-dissipative schemes:

- $c_x = 0.5c$, $c_y = 0.5c$
- $\alpha = 0.5c$

5. A hybrid differencing scheme is applied to discretize the advective flux and a CD scheme is applied to discretize the diffusive flux in 2D-convective-diffusion equations. The relevant parameters are $\alpha = 0.5$, $\tilde{a} = 0.5$, and $\tilde{b} = 0.5$. The coefficients $c_x$ and $c_y$ for any first-order convective-dissipative schemes:

- $c_x = 0.5c$, $c_y = 0.5c$
- $\alpha = 0.5c$

6. An assessment differencing scheme is applied to discretize the advective flux and a CD scheme is applied to discretize the diffusive flux in 2D-convective-diffusion equations. The relevant parameters are $\alpha = 0.5$, $\tilde{a} = 0.5$, and $\tilde{b} = 0.5$. The coefficients $c_x$ and $c_y$ for any first-order convective-dissipative schemes:

- $c_x = 0.5c$, $c_y = 0.5c$
- $\alpha = 0.5c$

7. In the NVER scheme, the value of $G$ at cell interfaces $\Gamma$ of a control volume $V$ is approximated as follows:

- $G_{\Gamma} = G_{\Gamma}^{\text{source}}$

8. A parameter $\theta$ is defined as $\theta = \frac{c}{c_v}$. For diffusive schemes, the flux-limiter function $\phi$ is defined as:

- $\phi(\theta) = \frac{\theta}{\theta + 1}$

9. For all advection-diffusion equations (ADE), demonstrate using the NVER scheme for advection terms and a hybrid differencing scheme (HDS) for diffusive terms, the expression:

- $G_{\Gamma} = G_{\Gamma}^{\text{source}}$

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