Assignment 8

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment. 

Due on 2019-03-27, 23:59 IST

1) In the number operator basis, the basis state expansion of the operator \( \hat{a}^3 = \sum_{n,m=0}^{\infty} d_{nm} |n\rangle \langle m| \) where \( d_{nm} \) will be

\[
\sqrt{n(n-1)\delta_{n-1,m}} \\
\sqrt{n(n-1)(n-2)\delta_{n-3,m}} \\
\sqrt{n(n+1)\delta_{n+2,m}} \\
\sqrt{m(m-1)(m-2)\delta_{n,m-3}}
\]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\( \sqrt{m(m-1)(m-2)\delta_{n,m-3}} \)

2) Consider a positively charged particle with charge \( q \) and mass \( m \) executing one-dimensional simple harmonic motion, with angular frequency \( \omega \), in a constant electric field background \( \mathcal{E} \). For such a system

Virial theorem implies that \( |n\rangle \) must be a simultaneous eigenstate of both kinetic energy operator \( \hat{T} \) and potential energy operator \( \hat{V}(\hat{x}) \)

ground state energy is \( \frac{1}{2} \hbar \omega \)

the ground state wavefunction does not have a maximum peak at \( x = 0 \).
3) The probability of finding the particle in the coherent state $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} e^{i\alpha d} |0\rangle$ in state $|n\rangle$ gives Poisson distribution.

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers: True

4) The expectation value of the operator $(\hat{a}^\dagger)^3 \hat{a}^3$ in any arbitrary state $|\psi\rangle$ will be always

- a negative complex number
- any real number
- a non-negative real number
- only an imaginary number

No, the answer is incorrect.
Score: 0
Accepted Answers: a non-negative real number

5) Consider a one dimensional harmonic oscillator in a superposed state with non-zero probability in ground and first excited state. The form of such a state for $\langle \hat{x} \rangle$ to be as large as possible will be

- $\frac{1}{\sqrt{2}} (|0\rangle + e^{i\alpha} |1\rangle)$ with $\alpha = \pi / 2$
- $\alpha (|0\rangle + \frac{\sqrt{1-\alpha^2}}{\alpha} |1\rangle)$ with $\alpha = 1 / \sqrt{2}$
- $\frac{1}{\sqrt{2}} (|0\rangle - e^{i\alpha} |1\rangle)$ with $\alpha = \pi / 2$
- $\frac{\alpha}{\sqrt{2}} (|0\rangle + \frac{\sqrt{1-\alpha^2}}{\alpha})$ with $\alpha = 1 / 2$

No, the answer is incorrect.
Score: 0
Accepted Answers: $\alpha (|0\rangle + \frac{\sqrt{1-\alpha^2}}{\alpha} |1\rangle)$ with $\alpha = 1 / \sqrt{2}$

6) Consider one dimensional simple harmonic oscillator in the ground state with $\hat{x}$ as the position operator. Then the quantity $\langle 0 | e^{i\lambda \hat{x}} | 0 \rangle$ will be

- $e^{-k^2 / 2} \langle 0 | \hat{x}^2 | 0 \rangle / 2$
- $e^{-k} \langle 0 | \hat{x} | 0 \rangle / 2$
- $0$
- $e^{k^2} \langle 0 | \hat{x}^2 | 0 \rangle$

No, the answer is incorrect.
Score: 0
Accepted Answers: $e^{-k^2 / 2} \langle 0 | \hat{x}^2 | 0 \rangle / 2$
A coherent state $|\alpha\rangle$ is defined as an eigenstate of ladder operator $\hat{a}$ (called annihilation operator) with eigenvalue $\alpha$ whose explicit form is $|\alpha\rangle = e^{-|\alpha|^2/2}e^{\alpha\hat{a}^\dagger}|0\rangle$. If $\hat{N}$ is the number operator then, $\langle\hat{N}\rangle$ gives $|\alpha|^2/2$.

The displacement operator $\hat{D}(\alpha) = e^{(\alpha\hat{a}^\dagger - \alpha^*\hat{a})}$ is unitary.

The correlation function $C(t) = \langle 0|\hat{x}(t)\hat{x}(0)|0\rangle$ with $\hat{x}(t)$ as the position operator in Heisenberg picture and $|0\rangle$ as ground state of the one-dimensional harmonic oscillator will be $\frac{4m}{\hbar} e^{-i\omega t}$.

$\hat{D}(\alpha) = e^{(\alpha\hat{a}^\dagger - \alpha^*\hat{a})}$ is the displacement operator. The quantity $\hat{D}^\dagger(\alpha)\hat{a}\hat{D}(\alpha)$ will be $\hat{a}^\dagger + \alpha$. 
The average energy of the harmonic oscillator in the coherent state $|\alpha\rangle$ is $\hbar \omega |\alpha|^2$.

- True
- False

11. No, the answer is incorrect.
Score: 0
Accepted Answers: False