Unit 8 - Week 7

Assignment 7

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment. Due on 2019-03-20, 23:59 IST.

1) \( Y_{\ell m}(\theta, \phi) \) will be a simultaneous eigenstate of

- \( \hat{L}_x, \hat{L}_y, \hat{L}_z \)
- \( \hat{L}^2, \hat{L}_z \)
- \( \hat{L}_z, \hat{L}_y, \hat{L}^2 \)
- \( \hat{L}_x, \hat{L}_z, \hat{L}^2 \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
- \( \hat{L}^2, \hat{L}_z \)

2) Selection rule for a transition from energy level \( n \) to energy level \( m \) to occur is

- \( \Delta \ell = \pm 1 \)
- \( \Delta m_\ell = \pm 1, 0 \)
- (a) or (b)
- (a) and (b)

No, the answer is incorrect.
Score: 0
Accepted Answers:
- (a) and (b)
4) If number operator \( \hat{\mathbf{N}} = \hat{\mathbf{a}}^{\dagger} \hat{\mathbf{a}} \) is operated on the state \(|n\rangle\) the eigenvalue will be

- 0
- \( n - 1 \)
- \( n \)
- \( n + 1 \)

No, the answer is incorrect.

Score: 0

Accepted Answers:
- \( n \)

5) Suppose Hamiltonian is \( \hat{\mathbf{H}} = \hbar \omega (\hat{\mathbf{a}}^{\dagger} \hat{\mathbf{a}} + 1/2) \) for a system, where the operators \( \hat{\mathbf{a}} \) and \( \hat{\mathbf{a}}^{\dagger} \) have nothing to do with the harmonic oscillator operators but are operators which satisfy anticommutation rules

\[
\{ \hat{\mathbf{a}}, \hat{\mathbf{a}}^{\dagger} \} = 1 \quad ; \quad \{ \hat{\mathbf{a}}, \hat{\mathbf{a}} \} = 0 \quad ; \quad \{ \hat{\mathbf{a}}^{\dagger}, \hat{\mathbf{a}}^{\dagger} \} = 0
\]

The energy eigenvalues for this system will be

- \( \frac{1}{2} \hbar \omega \) and \( \frac{3}{2} \hbar \omega \)
- \( \hbar \omega \) and \( 2 \hbar \omega \)
- \( 0 \) and \( \hbar \omega \)
- \( \frac{1}{2} \hbar \omega \) and \( \hbar \omega \)

No, the answer is incorrect.

Score: 0

Accepted Answers:
- \( \frac{1}{2} \hbar \omega \) and \( \frac{3}{2} \hbar \omega \)

6) Consider a four-dimensional linear vector space whose orthonormal basis states are \(|\alpha\rangle, |\alpha\rangle, |\gamma\rangle, |\delta\rangle\). The total number of allowed configurations for a system of two identical electrons in this vector space is

- 3
- 1
- 6
- 12

No, the answer is incorrect.

Score: 0

Accepted Answers:
- 6
7) The x-component of electron spin measurement gives $+\frac{1}{2}\hbar$. On this resulting state (which we will call $|\psi(t_0)\rangle$) we apply $\hat{H} = \frac{e}{mc} \hat{S}_z B$, where $B$ is the magnitude of the magnetic field along the $z$-direction. The state $|\psi(t)\rangle$ will be

\[
\frac{1}{2} \left[ e^{-iω(t-t_0)} |\uparrow\rangle - e^{iω(t-t_0)} |\downarrow\rangle \right]
\]

\[
\frac{1}{\sqrt{2}} \left[ e^{-iω(t-t_0)} |\uparrow\rangle - e^{iω(t-t_0)} |\downarrow\rangle \right]
\]

\[
\frac{1}{2} \left[ e^{-iω(t-t_0)} |\uparrow\rangle + e^{iω(t-t_0)} |\downarrow\rangle \right]
\]

\[
\frac{1}{\sqrt{2}} \left[ e^{-iω(t-t_0)} |\uparrow\rangle + e^{iω(t-t_0)} |\downarrow\rangle \right]
\]

No, the answer is incorrect.
Score: 0
Accepted Answers:

\[
\frac{1}{\sqrt{2}} \left[ e^{-iω(t-t_0)} |\uparrow\rangle + e^{iω(t-t_0)} |\downarrow\rangle \right]
\]

8) The number state $|0\rangle$ is the ground-state of the harmonic oscillator which is annihilated by $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \left( \hat{x} + \frac{i}{m\omega} \hat{p} \right)$ operator. The position-space wavefunction $\psi_0(x)$ will be

\[
A e^{-mωx^2/\hbar}
\]

\[
A e^{-x^2/mω}
\]

\[
A e^{-\sqrt{\frac{mω}{\hbar}} x^2}
\]

\[
A e^{\sqrt{\frac{mω}{\hbar}} x^2}
\]

No, the answer is incorrect.
Score: 0
Accepted Answers:

\[
A e^{-mωx^2/\hbar}
\]

9) The eigenvalues of Hamiltonian operator $\hat{H} = \epsilon(\sigma, \vec{n})$ (where $\epsilon$ is a constant, $\sigma$'s are Pauli matrices and $\vec{n}$ is a unit vector in an arbitrary direction) are

\[+\epsilon, -\epsilon\]

\[0, \epsilon\]

\[0, i\epsilon\]

\[-i\epsilon, i\epsilon\]

No, the answer is incorrect.
Score: 0
Accepted Answers:

\[+\epsilon, -\epsilon\]
Consider two identical non-interacting spin zero boson of mass $m$ in a one dimensional box. The energy measurement of the system was found to be $E_0 = \frac{\pi^2 \hbar^2}{2mL^2}$. The normalized position space wavefunction for the system will be

$$\psi(\theta, \phi) = \left( \begin{array}{c} \sin(\frac{\pi}{L}x) \sin(\frac{2\pi}{L}y) + \frac{1}{2} \sin(\frac{3\pi}{L}x) \sin(\frac{\pi}{L}y) \\ \sqrt{\frac{2}{L}} \sin(\frac{\pi}{L}x) \sin(\frac{2\pi}{L}y) - \sqrt{\frac{2}{L}} \sin(\frac{2\pi}{L}x) \sin(\frac{\pi}{L}y) \\ \frac{1}{\sqrt{2}} \left[ \frac{2}{L} \sin(\frac{\pi}{L}x) \sin(\frac{2\pi}{L}y) + \frac{2}{L} \sin(\frac{2\pi}{L}x) \sin(\frac{\pi}{L}y) \right] \\ \frac{1}{\sqrt{2}} \left[ \frac{2}{L} \sin(\frac{\pi}{L}x) \sin(\frac{2\pi}{L}y) - \frac{2}{L} \sin(\frac{2\pi}{L}x) \sin(\frac{\pi}{L}y) \right] \end{array} \right).$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\frac{1}{\sqrt{2}} \left[ \frac{2}{L} \sin(\frac{\pi}{L}x) \sin(\frac{2\pi}{L}y) + \frac{2}{L} \sin(\frac{2\pi}{L}x) \sin(\frac{\pi}{L}y) \right]$$

11) A system is in the state $\psi(\theta, \phi) = \frac{1}{\sqrt{8}} Y_{1,-1}(\theta, \phi) + \sqrt{\frac{3}{5}} Y_{1,0}(\theta, \phi) + \frac{1}{\sqrt{5}} Y_{1,1}(\theta, \phi)$. The quantity $\langle \psi | L_z | \psi \rangle$ will be

$$2\sqrt{\frac{6}{5}}, \quad \sqrt{\frac{3}{5}}, \quad \frac{2\sqrt{3}}{5}, \quad \sqrt{\frac{2}{5}}$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$2\sqrt{\frac{6}{5}}$$

12) If a system is in the state $\psi(\theta, \phi) = \frac{1}{\sqrt{5}} Y_{1,-1}(\theta, \phi) + \sqrt{\frac{3}{5}} Y_{1,0}(\theta, \phi) - \frac{1}{\sqrt{5}} Y_{1,1}(\theta, \phi)$ and $L_z$ is measured then the probabilities of getting the measured values with $m = 0$ will be

$$\frac{3}{5}, \quad \frac{1}{\sqrt{3}}, \quad \frac{1}{5}, \quad \sqrt{\frac{3}{5}}$$
No, the answer is incorrect.
Score: 0
Accepted Answers: 
3
5