Assignment 9

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment. Due on 2019-04-03, 23:59 IST.

1) Using the properties of the Pauli matrices, the expression \(\exp\left(-\frac{i\pi}{2}\right)\sigma_z \exp\left(-\frac{i\pi}{2}\right)\) turns out to be

- equal to \(\sigma_z + \pi\)
- proportional to \(\sigma_x\)
- proportional to \(\sigma_z\)
- proportional to \(\sigma_x + i\sigma_y\).

No, the answer is incorrect.
Score: 0
Accepted Answers:
- proportional to \(\sigma_z\)

2) The quantum mechanical form of the Lorentz force \(\vec{F} = e|\vec{E} + \vec{v} \times \vec{B}|\) is

- \(\frac{1}{2}e\vec{E}\) when \(\vec{B} = 0\)
- \(\frac{1}{2}e\left(\vec{v} \times \vec{B} + \vec{B} \times \vec{v}\right)\) when \(\vec{E} = 0\)
- \(e\vec{E} + \frac{3}{2}e\left(\vec{v} \times \vec{B} - \vec{B} \times \vec{v}\right)\)
- \(\frac{1}{2}e\left(\vec{v} \times \vec{B} - \vec{B} \times \vec{v}\right)\) when \(\vec{E} = 0\)
is \( \hat{H} = \frac{\alpha}{\hbar}(S_x^2 + S_y^2 - S_z^2) - \frac{\beta}{\hbar}S_z \) where \( \alpha \) and \( \beta \) are constants. The energy levels for such a particle with magnetic quantum number \( m = 3/2 \) will be

\[
\begin{align*}
-\frac{3\alpha}{4} + \frac{3\beta}{2} \\
-\frac{3\alpha}{4} - \frac{3\beta}{2} \\
\frac{13\alpha}{4} + \frac{3\beta}{2} \\
\frac{13\alpha}{4} - \frac{3\beta}{2}
\end{align*}
\]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\(-\frac{3\alpha}{4} - \frac{3\beta}{2}\)

4) In classical mechanics, we write an operator \( \Omega = \vec{p} \times \vec{L} \) where \( L \) is the angular momentum. Its operator form in quantum mechanics will be

\[
\begin{align*}
\vec{p} \times \hat{\vec{L}} \\
-\hat{\vec{L}} \times \vec{p} \\
\frac{1}{2} \left( \vec{p} \times \hat{\vec{L}} - \hat{\vec{L}} \times \vec{p} \right) \\
\frac{1}{2} \left( \vec{p} \times \hat{\vec{L}} + \hat{\vec{L}} \times \vec{p} \right)
\end{align*}
\]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\(\frac{1}{2} \left( \vec{p} \times \hat{\vec{L}} - \hat{\vec{L}} \times \vec{p} \right)\)

5) Consider an electron subjected to a uniform magnetic field \( \vec{B} = B\hat{e}_z \), with the vector potential \( \vec{A} \) and the Hamiltonian of the electron is \( \hat{H} = \frac{1}{2m}(\Pi_x^2 + \Pi_y^2 + p_z^2) \) where \( \Pi_i = p_i - \frac{eA_i}{c} \). The commutator bracket \([\Pi_x^2, \Pi_y]\) will be

\[
\begin{align*}
2\Pi_x \\
\frac{2i\hbar eB \Pi_y}{c} \\
p_z \\
\frac{2i\hbar eB \Pi_x}{c}
\end{align*}
\]

No, the answer is incorrect.
Score: 0
6) Runge-Lenz vector $\mathbf{A} = \frac{1}{2m_e c^2} (\mathbf{p} \times \mathbf{L} - \mathbf{L} \times \mathbf{p}) - \frac{e^2}{r}$ is an additional constant of motion for hydrogen atom described by Coulomb potential. All the degenerate wavefunctions of the first excited state ($n=2$) of the hydrogen atom (Here $\hat{H}$ is the hydrogen atom Hamiltonian, $\mathcal{P}$ is the parity and $\mathbf{L}$ denotes angular momentum operator.)

- [ ] have same parity eigenvalue because $[\hat{H}, \mathcal{P}] = 0$.

- [ ] some wavefunctions have odd parity eigenvalue and others have even parity eigenvalues because $[\hat{H}, \mathcal{P}] = 0$ and $[\mathcal{P}, \mathbf{A}] \neq 0$.

- [ ] are simultaneous eigenstates of both $\hat{L}^2, \mathcal{P}, \hat{A}_z$.

- [ ] are simultaneous eigenstates of $\hat{L}^2, L_z, \mathcal{P}$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
- some wavefunctions have odd parity eigenvalue and others have even parity eigenvalues because $[\hat{H}, \mathcal{P}] = 0$ and $[\mathcal{P}, \mathbf{A}] \neq 0$.
- are simultaneous eigenstates of $\hat{L}^2, L_z, \mathcal{P}$.

7) In classical mechanics, the radial component of the momentum is $p_r = \frac{\hat{r}}{|\mathbf{r}|} \cdot \mathbf{p}$. The quantum mechanical differential operator form in position basis is

- $\hat{r} p_r \hat{p}$

- $\mathbf{p} \cdot \frac{\hat{r}}{|\mathbf{r}|}$

- $\frac{\mathbf{p}}{|\mathbf{r}|} \cdot \hat{p} - \hat{p} \cdot \frac{\hat{r}}{|\mathbf{r}|}$

- $\frac{1}{2} \left( \frac{\hat{r}}{|\mathbf{r}|} \cdot \hat{p} + \hat{p} \cdot \frac{\hat{r}}{|\mathbf{r}|} \right)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
- $\frac{1}{2} \left( \frac{\hat{r}}{|\mathbf{r}|} \cdot \hat{p} + \hat{p} \cdot \frac{\hat{r}}{|\mathbf{r}|} \right)$

8) Consider a particle in a Coulomb potential in three dimensional space. In Heisenberg’s picture, $\frac{d}{dt} \langle \hat{L}_z \rangle = 0$, where $\hat{L}_z$ is the $z$-component of angular momentum operator. Using this information which of the following choice(s) are correct:

- $\hat{H}, \hat{L}_z, \hat{L}_x$ share simultaneous eigenbasis

- $\hat{L}_z, \hat{L}_x$ share simultaneous eigenbasis
9) Consider a spin operator in the x-z plane making an angle $\theta$ with the z-axis so that:

$$\hat{S}_n = \frac{\hbar}{2}(\sigma_z \cos \theta + \sigma_x \sin \theta)$$

where $\sigma_x$ and $\sigma_z$ are Pauli matrices. If the eigenvectors of $\hat{S}_z$ operator is:

$$\hat{S}_z |z, \pm \rangle = \pm \frac{\hbar}{2}|z, \pm \rangle$$

then the state $|n, \pm \rangle$ will be (n is in x-z plane)

- $\cos \theta |z, + \rangle + \sin \theta |z, - \rangle$
- $\cos \frac{\theta}{2} |z, + \rangle + \sin \frac{\theta}{2} |z, - \rangle$
- $\cos \theta |z, + \rangle - \sin \theta |z, - \rangle$
- $\cos \frac{\theta}{2} |z, + \rangle - \sin \frac{\theta}{2} |z, - \rangle$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\cos \theta |z, + \rangle + \sin \theta |z, - \rangle$

10) A spin 1 particle has Hamiltonian $\hat{H} = \hat{S}_z + \frac{2}{\hbar}\hat{S}_x^2$, then the energy eigenvalues of this particle will be:

- $2\hbar, \hbar, -\hbar$
- $\sqrt{2}\hbar, -\sqrt{2}\hbar, 2\hbar$
- $2\hbar, \hbar, \hbar$
- $2\hbar, (1 + \sqrt{2})\hbar, (1 - \sqrt{2})\hbar$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$2\hbar, (1 + \sqrt{2})\hbar, (1 - \sqrt{2})\hbar$