Assignment 8

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment. Due on 2019-03-27, 23:59 IST.

1) Let \( \mathbb{R}_2[t] \) be the vector space of all real polynomials of degree at most 2, with the inner product
\[
(p, q) = \int_{-1}^{1} p(t)q(t) \, dt.
\]
Check that \( B = \left\{ \frac{1}{\sqrt{2}}, \frac{\sqrt{6}}{2}, \frac{\sqrt{10}}{4}(3t^2 - 1) \right\} \) is an orthonormal basis. The coordinate vector of \( p(t) = 1 + t + t^2 \) with respect to the basis \( B \) is

\[
\begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{\sqrt{6}}{2} & \frac{\sqrt{10}}{4}
\end{pmatrix}
\]

No, the answer is incorrect.

Score: 0

Accepted Answers:
\[
\begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{\sqrt{10}}{4}
\end{pmatrix}
\]

2) For \( \alpha, \beta \in \mathbb{R} \), define
\[
A_\alpha = \begin{bmatrix}
\alpha & 0 \\
1 & 1
\end{bmatrix}, \quad B_\beta = \begin{bmatrix}
1.5 & \beta \\
-\frac{1}{2} & \beta
\end{bmatrix}
\]
be real matrices. Let \( U = \{ \alpha \in \mathbb{R} : A_\alpha \text{ is an orthogonal matrix} \} \) and \( V = \{ \beta \in \mathbb{R} : B_\beta \text{ is an orthogonal matrix} \} \). Then

\[
U = \left\{ \alpha \in \mathbb{R} : \alpha \right\}, \quad V = \left\{ \beta \in \mathbb{R} : \beta \right\}
\]
Let \( u = \{ -\frac{1}{2}, \frac{1}{2} \} \) and \( v = \{-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\} \).

No, the answer is incorrect.

Score: 0

Accepted Answers:
- \( U = \emptyset, \ V = \{-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\}\) (1 point)

Let \( u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} \in \mathbb{R}^n \) be a unit vector with \( u_1 \neq -1 \). Let us write \( u = \begin{bmatrix} u_1 \\ v \end{bmatrix} \),

where \( v = \begin{bmatrix} u_2 \\ \vdots \\ u_n \end{bmatrix} \in \mathbb{R}^{n-1} \). Let

\[
P = \begin{bmatrix} u_1 \\ v \\ -I_{n-1} + \frac{1}{1+u_1 v} v v^t \end{bmatrix},
\]

where \( I_{n-1} \) is the \((n-1) \times (n-1)\) identity matrix. Then \( P \) defines a linear map from \( \mathbb{R}^n \to \mathbb{R}^n \), which is

- invertible but not an isometry.
- invertible but not diagonalizable.
- invertible and diagonalizable.
- invertible and an isometry.

No, the answer is incorrect.

Score: 0

Accepted Answers:
- invertible and an isometry.

4) Recall that an \( n \times n \) matrix \( A = [a_{ij}] \) is said to be

1 point

1. upper triangular if \( a_{ij} = 0 \), for all \( i > j \).
2. lower triangular if \( a_{ij} = 0 \), for all \( i < j \).
3. triangular if \( A \) is either upper triangular or lower triangular.
4. diagonal if \( a_{ij} = 0 \), for all \( i \neq j \).
5. anti-diagonal if \( a_{ij} = 0 \), for all \( i \neq n+1 - j \).

Now let \( A \) be an \( n \times n \) real triangular matrix that is orthogonal. Then

- \( A \) is symmetric.
- \( A \) is diagonal.
- \( A \) is skew-symmetric.
- \( A \) is anti-diagonal.

No, the answer is incorrect.

Score: 0

Accepted Answers:
- \( A \) is diagonal.