Assignment 6

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment. 

Due on 2019-03-13, 23:59 IST

1) This question begins with a discussion. Suppose $A$ is an $n \times n$ matrix that is diagonalizable. Let $D$ be a diagonal matrix and $P$ be an orthogonal matrix such that $P^tAP = D$. Since $P$ is orthogonal, we also have $P^t = P^{-1}$. So $P^{-1}AP = D$, which gives $A = PDP^{-1}$. Then note that $A^2 = (PDP^{-1})(PDP^{-1}) = PD^2P^{-1}$,

$A^3 = (PD^2P^{-1})(PDP^{-1})$

More generally, we get $A^n = PD^nP^{-1}$, for any $n \in \mathbb{N}$.

Now suppose $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$. Then the matrix $A^6$ is equal to

$\begin{bmatrix} 1 & 1 \\ 64 & 1028 \end{bmatrix}$

$\begin{bmatrix} -601 & 665 \\ -1330 & 1394 \end{bmatrix}$

$\begin{bmatrix} -384 & 283 \\ 676 & 1028 \end{bmatrix}$

$\begin{bmatrix} 441 & 256 \\ 674 & 1394 \end{bmatrix}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\begin{bmatrix} -601 & 665 \\ -1330 & 1394 \end{bmatrix}$
No, the answer is incorrect.
Score: 0
Accepted Answers:
\[ \frac{1}{3} \begin{bmatrix} 3^n + 1 & 3^n - 1 \\ 3^n - 1 & 3^n + 1 \end{bmatrix} \]

3) Let \( A = [a_{i,j}]_{n \times n} \) be a matrix. Define \( \tilde{A} = [-1]^{-j}a_{i,j}]_{n \times n} \). Suppose \( \lambda_1, \ldots, \lambda_k \) are the distinct eigenvalues of \( A \) with algebraic multiplicities \( m_1, \ldots, m_k \) respectively. Then

\[ -\lambda_1, \ldots, -\lambda_k \] are the distinct eigenvalues of \( \tilde{A} \) with algebraic multiplicities \( m_1, \ldots, m_k \) respectively.

\[ (-1)^m \lambda_1, \ldots, (-1)^m \lambda_k \] are the distinct eigenvalues of \( \tilde{A} \) with algebraic multiplicities \( m_1, \ldots, m_k \) respectively.

\[ |\lambda_1|, \ldots, |\lambda_k| \] are the distinct eigenvalues of \( \tilde{A} \) with algebraic multiplicities \( m_1, \ldots, m_k \) respectively.

\[ \lambda_1, \ldots, \lambda_k \] are the distinct eigenvalues of \( \tilde{A} \) with algebraic multiplicities \( m_1, \ldots, m_k \) respectively.

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[ \lambda_1, \ldots, \lambda_k \text{ are the distinct eigenvalues of } \tilde{A} \text{ with algebraic multiplicities } m_1, \ldots, m_k \text{ respectively} \]

4) Let \( A \) be an \( m \times n \) real matrix. For \( \mu, \delta \in \mathbb{R} \), define \( B_{\mu,\delta} = \mu A^t A + \delta AA^t \). 1 point

Then

\( B_{\mu,\delta} \) has real eigenvalues for all \( \mu, \delta \in \mathbb{R} \).

\( B_{\mu,\delta} \) has real eigenvalues for all \( \mu \geq 0, \delta \geq 0 \).

\( B_{\mu,\delta} \) has real eigenvalues exactly for all \( \mu \geq 0, \delta \leq 0 \).

\( B_{\mu,\delta} \) has real eigenvalues exactly for all \( \mu \in \mathbb{R} \) with \( \delta = -\mu \).

No, the answer is incorrect.
Score: 0
Accepted Answers:
\( B_{\mu,\delta} \text{ has real eigenvalues for all } \mu, \delta \in \mathbb{R} \).

5) Let \( f, g : \mathbb{R}^n \to \mathbb{R}^n \) be isometries. Then 1 point
6) Which of the following statements is true?

- Every real matrix that is diagonalizable over \( \mathbb{R} \) is invertible.
- Every real invertible matrix is diagonalizable over \( \mathbb{R} \).
- A real matrix with all eigenvalues distinct and nonzero is both diagonalizable over \( \mathbb{R} \) and invertible.
- A real matrix with all eigenvalues nonnegative is diagonalizable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
A real matrix with all eigenvalues distinct and nonzero is both diagonalizable over \( \mathbb{R} \) and invertible.

7) Let \( u, v \in \mathbb{R}^n \) be orthogonal vectors such that \( u + v, u - v \) are also orthogonal. Then

- \( u = v \).
- \( u = \pm v \).
- either \( u = 0 \) or \( v = 0 \).
- \( ||u|| = ||v|| \).

No, the answer is incorrect.
Score: 0
Accepted Answers:
\( ||u|| = ||v|| \).