Questions and answers for Module 3

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1 Questions

1. What is the normalization condition for a two particle Hilbert space?

2. Write the discrete wave functions for the particle in a box.

3. Write the expression for the discrete energy levels for a quantum harmonic oscillator.

4. Write the properties for P-space and Q-space.

5. Write the evolution equation for the density operator.
2 Answers

1. \[ \langle x'_1, x'_2 | x_1, x_2 \rangle = \delta(x_1 - x'_1) \delta(x_2 - x'_2). \]

2. \[ \psi_n(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n \pi x}{L} \right), \]
   when \( n \) is even and
   \[ \psi_n(x) = \sqrt{\frac{2}{L}} \cos \left( \frac{n \pi x}{L} \right), \]
   when \( n \) is odd.

3. \( E_n = \left( n + \frac{1}{2} \right) \hbar \omega. \)

4. \( P = P^\dagger \) and \( P^2 = P. \)

Let \( Q \) be the complementary projection operator of \( P \) which is defined as

\[ Q \equiv I - P. \]

\( Q \) also satisfies the following relations,

\[ Q = Q^\dagger \] and \( Q^2 = Q. \)

As \( Q \) always projects a state that is orthogonal to \( |\psi_0\rangle \) in the manifold of the Hilbert space, one can write

\[ Q |\psi\rangle = (I - P) |\psi\rangle, \]

\[ = |\psi\rangle - P |\psi\rangle, \]

\[ = |\psi\rangle - P (|\psi_0\rangle + |\phi\rangle), \]

\[ = |\psi\rangle - P |\psi_0\rangle - P |\phi\rangle. \]

5. \[ \frac{\partial \rho}{\partial t} = \frac{1}{i\hbar} [H, \rho]. \]