Lecture 29: State-Space Filters (Frequency Compensation and Tuning)
Review

- Effect of gain bandwidth product on State-space filter performance
- $Q$ enhancement due to gain bandwidth product

$$Q_{GB} ; \frac{Q_a}{1 - \Delta \phi Q_a}$$
Frequency Compensation

- Basic building blocks of active filters are second-order negative feedback systems.
- Second-order active filter consists of amplifier and two integrators which have frequency dependence.
- If the amplifier is internally compensated it may be modeled as a first order system as the second pole is normally beyond gain bandwidth product.

Non-inverting amplifier: \[
\frac{1 + K}{1 + \frac{(1 + K)s}{GB}}
\]

Inverting amplifier: \[
\frac{-K}{1 + \frac{(1 + K)s}{GB}}
\]
Frequency Compensation (contd.,)

Loop gain of these negative feedback systems is

\[ \frac{GB}{s} \left( \frac{1}{1 + K} \right) \]

Phase error in the amplifier at \( \omega_0 \) is

\[ - (1 + K) \frac{\omega_0}{GB} \]

Integrator

\[ \frac{- \omega_0}{s} \left( 1 + \frac{\omega_0}{GB} + \frac{s}{GB} \right) \]

Phase error in the integrator

\[ - \frac{\omega_0}{GB} \]
State-Space Filters

- Use two inverting integrators and one summing amplifier in a loop
- The cumulative phase error in the loop at $\omega_0$

\[
\frac{1}{\left(1 + \frac{s}{GB}\right)^2} \cdot \frac{-1}{1 + \frac{3s}{GB}} - \frac{5\omega_0}{GB}
\]
State-Space Filters (contd.,)

- Single-stage summing amplifier modified by replacing Op Amp by an Op Amp and a buffer
Input-output Relations

Single stage summing amplifier
\[
\frac{-1}{1 + \frac{3S}{GB}}
\]

Composite summing amplifier
\[
\frac{V_{o1}}{V_i} = \frac{-\left(1 + \frac{3S}{GB}\right)}{\left(1 + \frac{3S}{GB} + \frac{9S^2}{GB^2}\right)}; \quad -1
\]

Phase error becomes zero
\[
\frac{V_{o1}}{V_{o2}} = \left(1 + \frac{3S}{GB}\right) \text{ which has a positive phase (lead) error}
\]
Composite Integrator

\[ V_i \rightarrow C \rightarrow V_{o1} \rightarrow V_{o2} \]

\[ (\text{GB/s}) \]

\[ \frac{1}{1+(s/\text{GB})} \]
Composite Integrator (contd.,)

\[ \frac{V_{o1}}{V_i} = \frac{-\omega_0}{s} \left( 1 + \frac{s}{GB} \right)^2 \left( 1 + \frac{s}{GB} \right) ; \quad -\frac{\omega_0}{s} \]
Composite Integrator and Summer

![Composite Integrator and Summer Diagram]

GB/s

\(-\frac{1}{1 + (3s/GB)}\)
Composite Integrator and Summer (contd.,)

\[ \frac{V_{o1}}{V_i} = \frac{\omega_0}{s} \left( 1 + \frac{3s}{GB} \right) \frac{1 + \frac{s}{GB} + 3 \left( \frac{s}{GB} \right)^2}{1 + \frac{s}{GB} + \frac{2s}{GB}} \]
Compensated VCF (Ackerberg-Mossberg Circuit)

- Uses composite integrator-summer

\[ \frac{\omega_0}{s} \left( 1 + 3 \frac{s}{GB} \right) \]

\[ \frac{1 + 2 \frac{s}{GB} + 6 \left( \frac{s}{GB} \right)^2}{1 + 2 \frac{s}{GB} + 6 \left( \frac{s}{GB} \right)^2} \]
Uncompensated and compensated filters

Q=5
Switched capacitor filter

- Any filter using LC or RC has its pole-frequency $\omega_0 = \frac{1}{RC}$ or $\frac{1}{\sqrt{LC}}$
- Tolerance of components has great influence on the accuracy with which it is fixed.
- Resistors and capacitors have poor tolerance and large temperature coefficients in integrated circuits
- Ratios of capacitors or resistors have very good tolerance (one order of magnitude better than absolute values).
Switched Capacitors

\[ V_o = -\frac{1}{C_1} \int i_c dt \]

\[ = -\frac{1}{C_1 R_1} \int V_i dt \]
Switched capacitor replaces the $R_1$
Switched capacitor replaces the $R_1$ (contd.,)

The capacitor is connected to the input initially during the period $\phi_1$ of the clock 1. It collects a charge of $CV_i$. This charge gets transferred to the capacitor $C_1$ during the $\phi_2$ of the clock 2, when it gets connected to 2, the virtual ground.
Switched capacitor replaces the $R_1$ (contd.,)

Charge per unit time

\[ \text{current} = \frac{V_i}{R_{eq}} = \frac{C V_i}{T} \]

\[ R_{eq} = \frac{T}{C} \]

\[ \omega_0 = \frac{1}{R_1 C_1} = \left[ \frac{C}{C_1} \right] \frac{1}{T} = \left[ \frac{C}{C_1} \right] f_c \]

- $w_0$ of the filter is now dependent on the ratio of capacitors and clock frequency. Precision filters can now be realized in monolithic form.

- Frequency of the clock will have to be higher than $2f_{\text{max}}$ where $f_{\text{max}}$ is frequency of highest desirable signal in the input to the filter.
Features of Switched Capacitor Filters

- It is programmable filter as the clock frequency can be changed over a wide range.
- Ratio of capacitors have a tolerance one order of magnitude than the absolute values.
- Temperature coefficient of capacitances is very close to zero.
- Switches introduce switching noise into the entire system.
- Require additional analog filters for band limiting (pre-filters) and smoothening (post-filters).
- With supply voltage scaling down the switches become more leaky reducing the performance.
Biquad based switched capacitor filter is MF10

- Cost $ \sim 2 \text{ for more than 1 k units}
- Center frequency \( (f_0) = 2 \text{ Hz to 20 kHz} \)
- Clock frequency \( (f_c) = 10 \text{ Hz to 1 MHz} \) \( (f_0 : f_c : 1 : 50) \)
- GBW of the Op Amp = 2.5 MHz
- Slew Rate of Op Amp = 7 V/msec
Tuning of Filters

- Need for tuning
  - Key parameters characterizing filters are $Q$ (quality factor), $f_0$ (normalizing frequency) and $H_0$ (factor determining the gain at $f_0$)
  - $Q$ and $H_0$ are dimensionless quantities and are ratios of capacitances and resistances in active RC filters
  - $f_0$ is inversely proportional to resistance and capacitances
  - Precision $R_s$ and $C_s$ are necessary to have specified $f_0$
Tuning of Filters (contd.,)

- Values of R and C should be independent of temperature and time
- Resistances and capacitances in ICs have very poor tolerances
- Resistances in ICs have very high temperature dependency whereas the capacitances in ICs have acceptable temperature sensitivities
- Tuning becomes necessary to achieve the required specifications
Tuning of $f_0$

- R, C or R and C are adjusted to get the precise $f_0$
- Magnitude or phase of the filter at $f_0$ can be used for tuning
- As R and C values drift RC tuning is not the best choice
- Voltage controlled tuning is preferable
- Digitally programmable analog reconfigurable front-end and back-end filters
Voltage Controlled Filter (VCF)

\[ \omega_0 = \frac{V_C}{10RC} \]
General VCF

\[
\frac{V_o}{V_i} = \left( \frac{s^2 + \frac{s}{\omega_0} + \alpha_3}{\omega_0^2 + \frac{s}{\omega_0 Q} + 1} \right)
\]

<table>
<thead>
<tr>
<th>(a_1=H_0)</th>
<th>(a_2 = 0)</th>
<th>(a_3=0)</th>
<th>High-Pass</th>
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<tbody>
<tr>
<td>(a_1=0)</td>
<td>(a_2 = -H_0)</td>
<td>(a_3=0)</td>
<td>Band-Pass</td>
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<td>(a_1=0)</td>
<td>(a_2 = 0)</td>
<td>(a_3 = H_0)</td>
<td>Low-Pass</td>
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<tr>
<td>(a_1=H_0)</td>
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<td>(a_3 = H_0)</td>
<td>Band-Stop</td>
</tr>
<tr>
<td>(a_1=H_0)</td>
<td>(a_2 = -H_0/Q)</td>
<td>(a_3 = H_0)</td>
<td>All Pass</td>
</tr>
</tbody>
</table>
Manual Tuning of a second-order filter

- $f_0$ of the filter to be tuned to $f_{\text{ref}}$, specified frequency, by trial and error adjustment of $V_c$ to make $V_{av} = 0$
- Use an oscillator with frequency $f_{\text{ref}}$
Manual Tuning of a second-order filter (contd.,)

\[
\frac{1}{R_L C_L} = \omega_{\text{ref}}
\]

\[
V_o = V'_p \sin(\omega_{\text{ref}} t + \phi)
\]

\[
V_{av} = \frac{V_p V'_p}{20} \cos \phi
\]
Manual Tuning of a second-order filter (contd.,)

- Sensitivity of $V_{av}$ to changes in phase shift is maximum at $\theta = \theta/2$ at which $V_{av} = 0$
- In case of LP and HP filters phase shift at $f_0$ should be $\theta/2$ at which $V_{av} = 0$ when the filter is tuned to $f_{ref}$
- In case of BP and BS filters $V_{av}$ is maximum when tuned to $f_{ref}$
- As sensitivity is zero when $V_{av}$ is maximum, BP and BS filters must be tuned using the LP or HP outputs of the VCF
Example

- High-pass filter with $f_{\text{ref}} = 1 \text{ KHz}$; $R = 1 \text{ kW}$ and $C = 0.1 \text{ mF}$ in the VCF; $f_0 = 1.592(V_C/10)10^3 \text{ Hz}$; $R_L = 100 \text{ kW}$, $C_L = 1 \text{ mF}$
- When $V_{av} = 0$, $V_C = 6.3$
Multiplying DAC used as Multiplier

- DAC converts a digital input to an analog output if the input is a fixed analog voltage $V_{\text{ref}}$

  $$V_o = V_{\text{ref}} \sum_{i=0..n-1} b_i 2^{-i} \quad \text{where } b_i = 0 \text{ or } 1$$

- If $V_{\text{ref}}$ is variable then the device becomes multiplying DAC and can be used as a multiplier
Multiplying DAC used as Multiplier (contd.,)

- 12-bit DAC 7821 costs about $3.15 for >1k units where as the multiplier MPY 634 costs $13.25 for > 1k units
Conclusion