Semiconductor Optical Communication Components and Devices
Lecture 34: Speed of Photodiodes

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http://www.iitk.ac.in/ee/faculty/det_resume/utpal.html
Photodiode Speed Considerations

\[
\frac{\partial p}{\partial t} = - \frac{p - p_o}{\tau_p} + G - \left( \frac{1}{q} \right) \nabla \cdot \vec{J}_p \quad \frac{\partial n}{\partial t} = - \frac{n - n_o}{\tau_n} + G + \left( \frac{1}{q} \right) \nabla \cdot \vec{J}_n
\]

where

\[
J = q \mu_p pE - qD_p \frac{\partial p}{\partial x}
\quad \text{and} \quad
J = q \mu_n nE + qD_n \frac{\partial n}{\partial x}
\]

\[
J = J_p + J_n + \varepsilon_r \varepsilon_o \frac{\partial E}{\partial t}
\]

\[
G \text{ is the generation rate of } e^- - h^+ \text{ pairs}
\quad \text{and is therefore the generation rate for both } e^- \text{ and } h^+
\]

\[
\therefore G(x,t) = G_0(x) + G_1(x) e^{j\omega t}
\]

With \( \omega \), the modulation frequency of photon density, assume that for small signal \( G_1 \ll G_0 \) the excess carrier concentration follows the same form as that of the photon modulation characteristics.

\[
p(x,t) = p_o(x) + p_1(x) e^{j\omega t} \quad \text{and} \quad n(x,t) = n_o(x) + n_1(x) e^{j\omega t}
\]

There is no phase lag as RC time constant is not the dominant delay mechanism (assumed).
Photodiode Speed

Where \( v_n \) and \( v_p \) are the saturated velocities of electrons & holes, respectively and carrier generation concentration is:

\[
G'_1 = \alpha_{abs} \Phi_o e^{-\alpha_{abs}x}
\]

\( \Phi_o \) = incident photon flux \hspace{1cm} \( \alpha_{abs} \) = absorption coefficient

\[
J_{n1}(x) = q\Phi_o e^{-\left(\frac{j\omega}{v_n}x\right)} \left[ \int_0^x \alpha_{abs} e^{-\alpha_{abs}x} e^{\frac{j\omega x}{v_n}} dx \right] + C
\]

Average photon flux is considered

\[
J_{n1}(0) = 0, \text{ gives } C=0
\]

\[
J_{p1}(x) = q\Phi_o e^{-\left(\frac{j\omega}{v_p}x\right)} \left[ \int_{wd}^x \alpha_{abs} e^{-\alpha_{abs}x} e^{\frac{j\omega x}{v_p}} dx \right] + B
\]

where \( J_{p1}(0) = 0, \text{ gives } B=0 \)

Therefore, using the equations in the previous slide:

\[
j\omega \frac{J_{n1}}{v_n} = -qG'_1 - \frac{\partial J_{n1}}{\partial x} \hspace{1cm} \text{and} \hspace{1cm} -j\omega \frac{J_{p1}}{v_p} = -qG'_1 - \frac{\partial J_{p1}}{\partial x}
\]
Photodiode Speed (Contd.)

The transit times for electrons and holes, respectively, for an i-region thickness ‘W_d’ are:

\[ t_{r_n} = \frac{W_d}{v_n}, \quad t_{r_p} = \frac{W_d}{v_p} \]

\[ J_{n1}(x) = -q \Phi_o \alpha_{abs} \begin{bmatrix} j \left( \frac{\omega}{v_n} \right)^x \\ e^{\frac{\omega}{v_n}x} - e^{-\alpha_{abs}x} \\ \alpha_{abs} + j \left( \frac{\omega}{v_n} \right) \end{bmatrix} \]

\[ J_{p1}(x) = q \Phi_o \alpha_{abs} \begin{bmatrix} -\alpha_{abs}W_d + j \left( \frac{\omega}{v_p} \right)(W_d - x) \\ e \alpha_{abs} - j \left( \frac{\omega}{v_p} \right) \\ -e^{-\alpha_{abs}x} \end{bmatrix} \]
As \( J_{a.c} = J_{p1} + J_{n1} \) we have

\[
J_{a.c}(\omega) = q \Phi_0 \alpha_{ab} w_d \left[ \frac{e^{-\alpha_{ab}w_d} - 1}{\alpha_{ab} w_d (\alpha_{ab} w_d - j\omega t_r)} + \frac{e^{-\alpha_{ab}w_d} (e^{j\omega t_r} - 1)}{j\omega t_r (\alpha_{ab} w_d - j\omega t_r)} \right] + q \Phi_0 \alpha_{ab} w_d \left[ \frac{1 - e^{-j\omega t_e}}{j\omega t_e (\alpha_{ab} w_d + j\omega t_r)} + \frac{1 - e^{-\alpha_{ab}w_d}}{\alpha_{ab} w_d (\alpha_{ab} w_d + j\omega t_r)} \right]
\]

\( t_e = \frac{w_d}{v_{esat}} \) and \( t_r = \frac{w_d}{v_{hsat}} \)

\( v_h = v_{hsat} \quad v_e = v_{esat} \)

With \( \omega = 0 \), in the limit \( \omega \rightarrow 0 \) \( J_{a.c}(0) = 0 \)

Thus \( J_o(0) = q \Phi_0 \left\{ 1 - e^{-\alpha_{ab}w_d} \right\} \)
Response of (Vertical Photo-Diodes)

The internal photodiode frequency response then becomes:

\[
H(\omega) = \frac{I(\omega)}{I(0)} = \frac{\alpha_{\text{abs}}w_d}{1 - e^{-\alpha_{\text{abs}}w_d}} \left[ \left( 1 - e^{-\alpha_{\text{abs}}w_d} \right) - \frac{e^{-\alpha_{\text{abs}}w_d} \left( e^{j\omega t_{\text{re}}} - 1 \right)}{j\omega t_{\text{re}}} \right] \left/ \left( \alpha_{\text{abs}}w_d - j\omega t_{\text{re}} \right) \right.
\]

\[
- \frac{\alpha_{\text{abs}}w_d}{1 - e^{-\alpha_{\text{abs}}w_d}} \left[ \left( 1 - \frac{e^{-\alpha_{\text{abs}}w_d}}{\alpha_{\text{abs}}w_d} \right) - \frac{\left( e^{j\omega t_{\text{rh}}} - 1 \right)}{j\omega t_{\text{rh}}} \right] \left/ \left( \alpha_{\text{abs}}w_d + j\omega t_{\text{rh}} \right) \right.
\]

\[
R(f) = \frac{R(0)}{\left( 1 + 4\pi^2 f^2 \tau^2 \right)^{1/2}}
\]

\[\alpha_{\text{abs}}, t_{r_p}, t_{r_n}\] are variables which decide the frequency response for a particular PIN structure.

\[
\Delta P_{\text{electric}} \ (\text{dB}) = 10 \log \frac{P_{\text{electric}}}{P_{\text{electric}_i}} = 20 \log \frac{P_{\text{optical}}}{P_{\text{optical}_i}}
\]

\[
\Delta P_{\text{electric}} \ (\text{dB}) = 2 \Delta P_{\text{optical}} \ (\text{dB})
\]
Load Photocurrent frequency response

\[ I_L(\omega) = \frac{R_D}{K_1 + j\omega K_2 - \omega^2 K_3 - j\omega^3 K_4} \]

- \( R_D = 50 \text{M} \), \( R_S = 10 \), \( L_S = 60 \text{pH} \), \( C_{jnc} = 80 \text{fF} \), \( C_p = 15 \text{fF} \), \( R_L = 50 \)

1. If the diode is RC-time constant limited then area needs to be reduced to make it faster ---- Back Illumination necessary.

2. To work at faster speeds one needs to make the device materials dependent.

------ Transit time limited (electrons and holes need to travel a distance ‘\( W_d \)’ at a saturated velocity ‘\( v_s \)’ to be collected).
**Epitaxial Layer Design of the RCE P-I-N Photodetector**

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness (mm)</th>
<th>Doping (cm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>InGaAs</td>
<td>30</td>
<td>p⁺ 10¹⁹</td>
</tr>
<tr>
<td>Graded Layer</td>
<td>30</td>
<td>p⁺ 10¹⁹</td>
</tr>
<tr>
<td>InAlAs</td>
<td>210</td>
<td>p⁺ 10¹⁹</td>
</tr>
<tr>
<td>InAlAs</td>
<td>50</td>
<td>n⁻ 10¹⁶</td>
</tr>
<tr>
<td>Graded Layer</td>
<td>30</td>
<td>n⁻ 10¹⁶</td>
</tr>
<tr>
<td>InGaAs</td>
<td>300</td>
<td>n⁻ 10¹⁶</td>
</tr>
<tr>
<td>Graded Layer</td>
<td>30</td>
<td>n⁻ 10¹⁶</td>
</tr>
<tr>
<td>InAlAs</td>
<td>60</td>
<td>n⁻ 10¹⁶</td>
</tr>
<tr>
<td>InAlAs</td>
<td>300</td>
<td>n⁺ 3x10¹⁸</td>
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<tr>
<td>InAlAs</td>
<td>240</td>
<td>None</td>
</tr>
<tr>
<td>25 pair InAlAs/InAlGaAs DBR</td>
<td>25x(121/11 2)</td>
<td>None</td>
</tr>
<tr>
<td>InP Substrate</td>
<td>600pm</td>
<td>Semi-insulating</td>
</tr>
</tbody>
</table>

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Review Questions

1. In prob. 1 of lec. 34 if the width of the absorption region is increased to 2.0\(\mu\)m, how is the above estimate in variance with the actual value. Determine the same by writing a small program.

2. What are the techniques for the speed measurement of fast photodiodes? If one measures the impulse response by a fs laser, how does one find the 3dB bandwidth of the photodiode from the measured response? How does one do a photodiode speed measurement when neither a laser can be modulated at the highest speed that the photodiode responds nor can an oscilloscope be found to respond to the speed of the detector? Do some research on it.

3. A communication link is driven by a \(\lambda_o=1.5\mu m\) single mode laser of linewidth 10nm, which has a 3dB modulation bandwidth of 31.8GHz and negligible chirping. The channel is a single mode dispersion shifted fiber with a dispersion of 0.5 ps/(Km.nm) at \(\lambda_o\). The front end of the receiver is a PIN photodiode whose response time is transit time limited to 10ps. What is the maximum length of the fiber for which 10 Gbits/s operation is possible?