Semiconductor Optical Communication Components and Devices

Lecture 33: Photodiode Temporal Response

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http://www.iitk.ac.in/ee/faculty/det_resume/utpal.html
Bandwidth for uniform absorption - I

In integration when several components are involved each ones delay counts and the total delay is given by:

$$\tau_{\text{Total}}^2 = \tau_1^2 + \tau_2^2 + \tau_3^2 + \ldots$$

The individual responses could be Sinc functions as:

$$\sin(\omega_m \tau_d / 2) / (\omega_m \tau_d / 2)$$

Let the pulses be Gaussian and Fourier transform of pulses is also Gaussian.
Let pulse be $g(t) = e^{-\sigma^2 t^2}$

at $g(t) = \frac{1}{2}$, $t = \pm \sqrt{\ln 2} / \sigma$

The Fourier transform of a time Gaussian pulse is then

$$G(f) = F[g(t)] = \int_{-\infty}^{+\infty} g(t) e^{-j \omega t} dt = \frac{\sqrt{\pi}}{\sigma} e^{-\omega^2 / 4\sigma^2}$$

FWHM of power

$$\frac{2}{\sigma} \sqrt{\ln 2}$$

3dB frequency of

$$\frac{\sqrt{\ln 2} \cdot \sigma}{\pi} = \text{BW}$$

BW x Pulsewidth product

$$\frac{2 \ln 2}{\pi} \approx 0.45$$

If the impulse is a triangular one

$$h(t) = 1 - \frac{1}{\tau} t, \quad 0 < t < \tau$$

$$\text{BW}_{3\text{dB}} = \frac{0.45}{\tau} \text{Hz} \quad \tau_d = \frac{W_d}{v_e}$$
Bandwidth for uniform absorption - II

\[ h(t) = \int_{-\infty}^{\infty} (1 - \frac{t}{\tau}) e^{-j\omega t} dt = \int_{0}^{\tau} (1 - \frac{t}{\tau}) e^{-j\omega t} dt = \int_{0}^{\tau} e^{-j\omega t} dt - \frac{1}{\tau} \int_{0}^{\tau} te^{-j\omega t} dt \]

\[ = \frac{-1}{j\omega} e^{-j\omega \tau} \left[ -\frac{1}{\tau} \left[ te^{-j\omega t} \right]_{0}^{\tau} - \int_{0}^{\tau} \frac{1}{j\omega} e^{-j\omega t} dt \right] = \frac{-j}{\omega} \left( 1 - e^{-j\omega \tau} \right) - \frac{j}{\omega} e^{-j\omega \tau} + \frac{j}{\omega \tau} (e^{-j\omega \tau} - 1) \]

\[ = \frac{j}{\omega} \left[ \frac{e^{-j\omega \tau/2} - e^{j\omega \tau/2}}{j\omega \tau} e^{-j\omega \tau/2} - 1 \right] \]

\[ = \frac{j}{\omega} \left[ -\cos(\omega \tau/2) - j\sin(\omega \tau/2) - \cos(\omega \tau/2) - j\sin(\omega \tau/2) - 1 \right] \]

The Fourier transform of this is \( H(\omega) = \frac{j}{\omega} \left( \frac{\sin \frac{\omega \tau}{2}}{\omega \tau/2} \exp \left( -j \frac{\omega \tau}{2} \right) - 1 \right) \)

The 3-dB bandwidth of this is \( B_{3dB} = \frac{0.55}{\tau} \) Hz

When \( \tau_d \) and \( \tau_{RC} \) are involved, approximately \( BW = \frac{\sqrt{3}}{2\pi \left( \tau_{RC}^2 + \tau_d^2 \right)^{1/2}} \)
Carrier Transit Time Effect (Exponential Profile)

But \( N_0 \) is not uniform as \( N(z) = N(0) e^{-az} \) & \( N_{e_0} = N_{h_0} = N_0 = \int_0^{W_d} N(z) dz \)

Let \( N_e(0) = N_h(0) = N_0 \)

or \( \frac{N_0}{2} = N(0) \left[ 1 - e^{-az} \right] \)

\[ \therefore i_e(t) = \frac{q I_o v_e}{h \nu W_d} e^{-\alpha W_d} \left\{ e^{-\alpha W_d (1-t/\tau_e)} - 1 \right\} \]

\[ 0 < t < \tau_e \]

\[ \therefore i_{ph}(0) = \frac{q I_o}{h \nu W_d} \left[ 1 - e^{-\alpha W_d} \right] \left[ v_e + v_h \right] \]

However Hole Velocity being much less than electron velocity like: \( v_e \sim 5v_h \) \( \therefore v_e > v_h \) has implications on the type of photodiode illumination for high speed operation as shown below.
**PIN:**
High velocity means high field but that may also translate to large dark current.

Dark current ($I_d$) leading to noise, Responsivity ($R$), and Bandwidth ($B$) leading to speed will have to be optimized.

What is now to be used is:

- $\alpha$-SiC (4H)
- Si
- $\beta$-SiC (6H)
- Diamond
- InP
- GaAs
- GaN
- AlGaN/GaN
- AlN

**Electron Saturated Velocity**

![Graph showing Electron Velocity vs Electric Field for various materials](Image)
Hole Saturated Velocity

PIN:
High velocity means high speed but hole velocities are an order lower than in compound semiconductors than Si, Ge.

Bias to be used such that there is no breakdown of the diode but the field is high enough to give:

\[
\begin{align*}
\text{Hole Velocities (cm/s)} & \\
\text{Electric field (V/cm)} & \\
10^5 & 10^6 & 10^7 & 10^3 & 10^4 & 10^5
\end{align*}
\]

\[
\begin{align*}
\alpha\text{-SiC} & \\
\beta\text{-SiC} & \\
\text{GaAs} & \\
\text{InP} & \\
\text{In}_{0.53}\text{Ga}_{0.47}\text{As} & \\
\text{Si} & \\
\text{Diamond} & \\
\end{align*}
\]
# Electron Mobility and Saturated Velocity

<table>
<thead>
<tr>
<th>Semi-conductor</th>
<th>Bandgap (eV)</th>
<th>Electron Mobility ($i \text{cm}^2/\text{V.s}$)</th>
<th>Saturation Velocity (cm/s)</th>
<th>Electric Field (V/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>1.11</td>
<td>1350</td>
<td>$v_e=10^7$</td>
<td>$E = 5 \times 10^4$</td>
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<tr>
<td>Si</td>
<td>1.11</td>
<td></td>
<td>$v_h=7 \times 10^6$</td>
<td>$E = 1 \times 10^5$</td>
</tr>
<tr>
<td>InP</td>
<td>1.34</td>
<td></td>
<td>$v_e=3 \times 10^7$</td>
<td>$E = 1 \times 10^4$</td>
</tr>
<tr>
<td>InP</td>
<td>1.34</td>
<td></td>
<td>$v_h=10^6$</td>
<td>$E = 1 \times 10^4$</td>
</tr>
<tr>
<td>GaAs</td>
<td>1.425</td>
<td>6000</td>
<td>$v_e=2 \times 10^7$</td>
<td>$E = 3 \times 10^3$</td>
</tr>
<tr>
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<td>1.425</td>
<td></td>
<td>$v_h=4 \times 10^6$</td>
<td>$E = 8 \times 10^4$</td>
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<tr>
<td>InGaAs</td>
<td>0.86</td>
<td>12000</td>
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</tr>
<tr>
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<td>100</td>
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<tr>
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<td>300</td>
<td>$v_e=1.4 \times 10^7$</td>
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<tr>
<td>AlN</td>
<td>3.2</td>
<td>14</td>
<td>$v_h=4.2 \times 10^6$</td>
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<tr>
<td>InGaN</td>
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<td>4000</td>
<td>$v_e=4.2 \times 10^7$</td>
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</tr>
<tr>
<td>SiC</td>
<td>3.2</td>
<td>800</td>
<td>$v_e=2 \times 10^7$</td>
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</tbody>
</table>
**Peak Versus Saturated Velocity**

PIN: High velocity at peak than saturated means high stability in bias required
1. Consider the detector of prob. 2, lec. 32. If the photons are incident from the P+ side and the detector is operated under saturated velocity of the electrons ($v_e=2.5 \times 10^5$ m/s) and holes ($v_h=5 \times 10^3$ m/s), estimate the response time of the detector if the junction-area of the diode is very small.

2. What composition of InGaAsP should be used for the detection of $\lambda=1.3\mu$m from speed considerations.

3. How should the a PIN Photodiode be biased for maximum speed of operation.